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Theory of wave scattering by distributed scatterers for remote sensing applications

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Abstract. We show that the Fraunhofer diffraction approximation can never be used in the theory of scattering by distributed scatterer (such as turbulence) in the far zone of antenna. We consider the single scattering of waves in a medium containing weak refractive index inhomogeneities using a Fresnel diffraction approximation. We do not introduce scattering volume but consider a transmitting antenna with a Gaussian distribution of current and a receiving antenna with a Gaussian distribution of attenuation across the aperture. The results obtained are valid both in the near and far zones of the antenna. The known contradiction between the applicability of the Fraunhofer diffraction approach and the size of the scattering volume formed by intersection of directivity diagrams is resolved. Similar to the far zone approach, the scattering intensity obtained is proportional to the Bragg component of the inhomogeneities that are averaged in Fourier space using some specific weighting function, depending on antenna size and pulse duration. The concept of a scattering cross section cannot be used for this problem and another more flexible normalization is suggested.

1 Introduction

The standard theory of electromagnetic wave scattering in a medium containing random inhomogeneities is based on the Fraunhofer diffraction approximation Tatarskii (1967). This means that in limits of transverse scale $L_\perp$ of the scatterer we can consider the incident wave as a plane wave. The condition of applicability of the Fraunhofer diffraction approximation is as follows: $L_\perp^2 \ll \lambda r$, where λ is the wavelength and r is the distance from the antenna to the scatterer. If this condition is true, we can neglect the quadratic terms in the phase expansion in powers of transverse coordinates. This condition may be fulfilled for a small isolated discrete scatterer. The situation is different if we consider wave scattering in a medium containing distributed random inhomogeneities. In this case, the scatterer (i.e., fluctuations of the refractive index) fill in the whole antenna beam and the transverse scale of the scatterer $L_\perp$ coincides with the antenna beam width D. If we determine the scattering volume to be the overlapping of the transmitting and receiving antenna diagrams, we obtain $D \sim \gamma r$, where γ is the angular size of antenna beam: $\gamma \sim \lambda/a$. Here, a is the antenna size (we do not include any numerical coefficients in formulae describing proportionality $\sim$). In the far zone of the antenna, where $r \gg a^2/\lambda$ or $\sqrt{\lambda r}/a \gg 1$, we have $D \approx (\lambda/a)\, r = \left(\sqrt{\lambda r}/a\right) \sqrt{\lambda r} \gg \sqrt{\lambda r}$. Because we have $L_\perp \approx D$ for a distributed scatterer, this inequality means that in actuality, in the far zone of the antenna, $L_\perp \gg \sqrt{\lambda r}$ and therefore the condition of applicability of the Fraunhofer diffraction, $L_\perp \ll \sqrt{\lambda r}$, cannot be satisfied for distributed scatter. This situation is illustrated in Figure 1. The antenna beam is shown in blue, the Fresnel zone is shown in red. The ticks along the abscissa represent the ratio $Z = r/ka^2$ (the distance from the antenna, measured in Fraunhofer length ka^2); the ticks along the ordinate represent the transverse scales of the beam and the Fresnel zone, measured in antenna scale a. Inhomogeneities are schematically shown in green, but all the collection of green objects must be considered as a single scatterer. In the far zone of the antenna (i.e., $Z \gg 1$) inhomogeneities fill in the whole incident beam, and because of this it is impossible to satisfy the condition $L_\perp \ll \sqrt{\lambda r}$.

The attempt to resolve this contradiction was made in Tatarskii (1971), in which the variable scattering vector that changed its direction inside the scattering volume was introduced. To derive the scattering cross section, the condition $L_0^2 \ll \lambda r$ was used, where L_0 is the correlation (integral) scale of inhomogeneities. However, condition $L_0^2 \ll \lambda r$ contains the scale L_0, which has no relation to the actual scattering Bragg scale component of dielectric permittivity fluctuations. We can, for instance, consider two inhomogeneous media having different scales L_0, but have the same intensity of the Bragg scale Fourier component. These two media would scatter the wave in a similar way, but condition $L_0^2 \ll \lambda r$ may be true for one of them but not for the other

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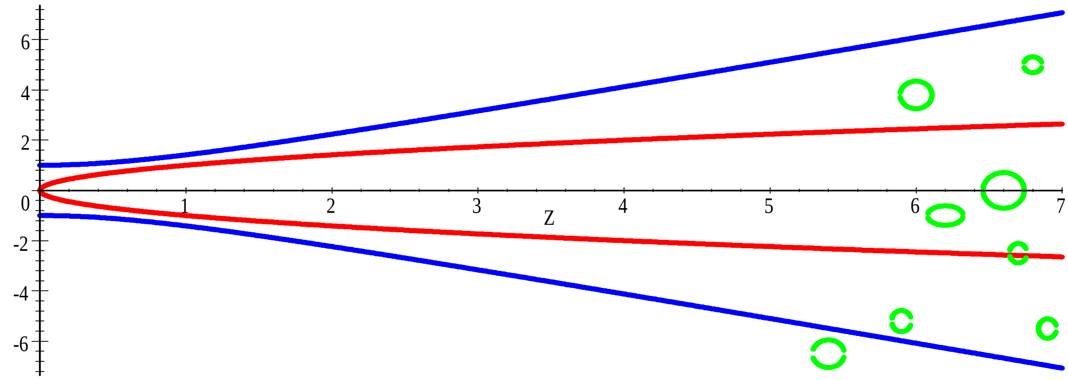

Fig. 1. In the case of random fluctuations of dielectric permittivity, the scatterer (instantaneous distribution of ε') fills in all the incident beam and its scale can not be considered as small in comparison with $\sqrt{\lambda r}$.

one. Thus, it is unlikely that condition $L_0^2 \ll \lambda r$ is really important.

In this paper, we consider the scattering in a more realistic formulation. To avoid usage of the condition $L_\perp^2 \ll \lambda r$, we consider the scattering in the Fresnel approximation. Instead of introducing the scattering volume, we include the model of transmitting and receiving antennas in an explicit form. Such a consideration allows us to avoid bringing in the scattering volume - the corresponding weighting function appears automatically. We consider these antennas as Gaussian (Gaussian distribution of the amplitude of initial field across the transmitting antenna aperture, and Gaussian attenuation across the receiving antenna aperture). Such an approach works well with the description of scattering in the Fresnel approximation, in which we keep the quadratic terms in the phase expansion. The combination of these two approximations allows one to perform all the calculations in closed analytical form. The longitudinal scale of the scattering volume is determined by the electromagnetic pulse envelope, which we also assume to be Gaussian. The correlation length L_0 does not appear among the parameters determining the applicability of the considered Fresnel scattering approach.

2 Single scattering in the Fresnel diffraction approximation

In the scalar approximation, the scattered field is given by the formula

$$E_{\mathrm{sc}}(\mathbf{r},t) = -k^2 \iiint G_0(\mathbf{r}-\mathbf{r}')\,\varepsilon'(\mathbf{r}')\,E_0(\mathbf{r}',t)\,d^3r'. \quad (1)$$

Here, $G_0(\mathbf{r}-\mathbf{r}') = -\exp[ik|\mathbf{r}-\mathbf{r}'|]/4\pi|\mathbf{r}-\mathbf{r}'|$, $\varepsilon'(\mathbf{r}')$ is the fluctuating part of dielectric permittivity, $E_0(\mathbf{r}',t)$ is the complex amplitude of the incident wave.

Before considering different approximations (Fraunhofer or Fresnel diffraction) we note that according to (1), the single scattered by dielectric permittivity fluctuations field is *linear* in $\varepsilon'(\mathbf{r})$. If we substitute $\varepsilon'(\mathbf{r}')$ in the form of the random Fourier integral

$$\varepsilon'(x',y',z')$$

$$= \iiint_{-\infty}^{\infty} \exp(ip_x x' + ip_y y' + ip_z z')\,\widetilde{\varepsilon}(\mathbf{p})\,d^3p, \quad (2)$$

we obtain the formula having the structure

$$E_{\mathrm{sc}}(\mathbf{r}) = \iiint U(\mathbf{r},\mathbf{p})\,\widetilde{\varepsilon}(\mathbf{p})\,d^3p. \quad (3)$$

This formula is precise and it is true for Fraunhofer, Fresnel, and all other possible more precise approximations, describing *single* scattering. Only the kernels $U(\mathbf{r},\mathbf{p})$ are different for different approximations. The intensity of the scattered field has the form

$$\langle E_{\mathrm{sc}}(\mathbf{r})\,E^*_{\mathrm{sc}}(\mathbf{r})\rangle$$
$$= \iiint d^3p' \iiint d^3p''\,U(\mathbf{r},\mathbf{p}')\,U^*(\mathbf{r},\mathbf{p}'')\,\langle\widetilde{\varepsilon}(\mathbf{p}')\,\widetilde{\varepsilon}^*(\mathbf{p}'')\rangle . \quad (4)$$

However, for a statistically homogeneous field ε' the random Fourier component $\widetilde{\varepsilon}(\mathbf{p})$ satisfies the relation (see, e.g. Rytov et al., 1989, vol. **3**, page 10, formula (1.47)), $\langle\,\widetilde{\varepsilon}(\mathbf{p})\,\widetilde{\varepsilon}^*(\mathbf{p}')\rangle = \Phi_\varepsilon(\mathbf{p})\,\delta(\mathbf{p}-\mathbf{p}')$, where $\Phi_\varepsilon(\mathbf{p}) = \Phi_\varepsilon(-\mathbf{p})$ is the power spectrum of the refractive index. Using this formula we obtain for the intensity of the scattered field Wheelon (1972):

$$\langle E_{\mathrm{sc}}(\mathbf{r})\,E^*_{\mathrm{sc}}(\mathbf{r})\rangle = \iiint \Phi_\varepsilon(\mathbf{p})\,|U(\mathbf{r},\mathbf{p})|^2\,d^3p. \quad (5)$$

The integrand is proportional to the product of the spectrum $\Phi_\varepsilon(\mathbf{p})$ by the weighting (window) function $|U(\mathbf{r},\mathbf{p})|^2$. The shape of this window function depends on the approximation used.

The window function $|U(\mathbf{r},\mathbf{p})|^2$ for the case of Fraunhofer scattering is known Tatarskii (1967): it is some narrow function centered around the Bragg wave vector and having the scales $1/L_x, 1/L_y$, and $1/L_z$, where L_x, L_y, and L_z are the scales of scattering volume V. We are not sure, however, that this result is correct, because of the described self-contradiction in the problem's formulation. Our goal is to determine this function for the case of Fresnel scattering.

Let us return to the scattered field, which is described by the formula (1). The incident field, $E_0(\mathbf{r}',t)$, assumed to be radiated by the transmitting antenna with Gaussian distribution of currents in the aperture plane, is described by the formula

$$E_0(\mathbf{r}',t) = A_0 M(\mathbf{r}',t) \iint_{-\infty}^{\infty} d\xi d\eta G_0(x'-\xi, y'-\eta, z') \exp\left(-\frac{\xi^2+\eta^2}{2a^2}\right). \tag{6}$$

Here, A_0 is the amplitude, $\mathbf{r}' = (x', y', z')$, and z'-axis is directed perpendicularly to the antenna aperture (not necessarily vertical). The function $M(\mathbf{r}',t)$ describes the time envelope of the radiated pulse. The vector $\overrightarrow{\rho} = (\xi, \eta, 0)$ lies in the plane of the antenna aperture. The value a describes an effective radius of the transmitting antenna.

In the near zone of the antenna ($z' \ll a^2/\lambda$), we may approximate the time envelope $M(\mathbf{r}',t)$ by the flat moving slab of the Gaussian form $M(\mathbf{r}',t) = \exp\left[-(z'-ct)^2/2h^2\right]$. In the far zone of the antenna, we can use the same formula if the condition $A \equiv \lambda^4 z'^2/8h^2a^4 \ll 1$ is true. For instance, if $\lambda = 0.3$ m, $a = 3$ m, $z' = 3000$ m, and $h = 30$ m, we obtain $A = 0.125$.

Using $M(\mathbf{r}',t) = \exp\left[-(z'-ct)^2/2h^2\right]$ in (6), we obtain

$$E_0(\mathbf{r}',t) = A_0 \exp\left[-\frac{(z'-ct)^2}{2h^2}\right] \iint_{-\infty}^{\infty} G_0(x'-\xi, y'-\eta, z') \exp\left(-\frac{\xi^2+\eta^2}{2a^2}\right) d\xi d\eta. \tag{7}$$

In this paper, we consider only the backscattering case and assume that the receiving antenna coincides with the transmitting antenna. The scattered field at the point $\mathbf{r}$ of the receiving aperture $E_{\text{sc}}(\mathbf{r},t)$ is determined by formulae (1) and (7). The field at the receiving antenna feed and the signal are proportional to the integral of $E_{\text{sc}}(\mathbf{r},t)$ over receiving aperture $z=0$:

$$\mathcal{E} = \iint E_{\text{sc}}(x,y,0,t) \exp\left(-\frac{x^2+y^2}{2a^2}\right) dxdy. \tag{8}$$

The distance z' from the antenna to the scattering volume is about $R = ct$, and we assume that $z' \gg L_\perp$. We use the expansion

$$|\mathbf{r}-\mathbf{r}'| = |z'| + \frac{(x-x')^2+(y-y')^2}{2|z'|} - \frac{\left[(x-x')^2+(y-y')^2\right]^2}{8|z'^3|} + \cdots. \tag{9}$$

and take into account quadratic terms in $\rho_\perp^2 = (x-x')^2 + (y-y')^2$. To neglect the last term in (9), the condition $k\rho_\perp^4/8z'^3 \ll 1$ must be true. This restriction is typical for a Fresnel diffraction approximation. It can be presented in the form $\rho_\perp^2 \ll \lambda R (\rho_\perp/R)^{-2}$, where $z' \approx R = ct$. Because of the relation $\gamma = \rho_\perp/R$ for the angular size γ of the antenna beam, we can present the last condition in the form

$$\rho_\perp \sim D \ll \frac{\sqrt{\lambda R}}{\gamma}. \tag{10}$$

If the numerical factor $\gamma \ll 1$, i.e., if the antenna beam is narrow enough, condition (10) can be true even if $D \gg \sqrt{\lambda R}$, i.e., in the far zone of the antenna. Thus, instead of the restriction $D \ll \sqrt{\lambda R}$, which cannot be satisfied in the far zone of antenna (see Figure 1), in the case of Fresnel approximation we obtain the restriction (10), which may be true in the far zone if $\gamma \ll 1$. Thus, if we retain the quadratic term in the phase expansion, we resolve the contradiction described in the Introduction. The relations between the beam width, Fresnel zone $\sqrt{\lambda R}$, and $\sqrt{\lambda R}/\gamma$ for $\gamma = 0.1$ is shown in Figure 2. It is clear that the condition (10) can be fulfilled in a wide range of distances.

Substituting (7), (8) in (1), and using (9) for G_0, (we retain only the first terms of this expansion in the denominators of Green's functions), we obtain some multidimensional integral that contains quadratic form in the exponent. This integral can be evaluated analytically for the case $k\rho_\perp^2 h/R^2 \ll 1$ and the result is given by the formula

$$\mathcal{E} = -\frac{k^2 A_0 a^4}{4(R-ika^2)^2} \iiint d^3r' \varepsilon'(\mathbf{r}') \exp\left[2ikz' - \frac{(z'-R)^2}{2h^2} + \frac{ik(x'^2+y'^2)}{R-ika^2}\right]. \tag{11}$$

Let us substitute $\varepsilon'(\mathbf{r}')$ in the form of the random Fourier integral (2). Changing the order of integration, we obtain:

$$\mathcal{E} = -\frac{\sqrt{2\pi}\pi ik A_0 a^4 h}{4(R-ika^2)} \iiint_{-\infty}^{\infty} \widetilde{\varepsilon}(\mathbf{p}) d^3p \exp\left[-\frac{iR}{4k}(p_x^2+p_y^2) + i(p_z+2k)R - \frac{a^2}{4}(p_x^2+p_y^2) - \frac{h^2(p_z+2k)^2}{2}\right]. \tag{12}$$

Let us consider the intensity $\langle \mathcal{E}\mathcal{E}^* \rangle$. Multiplying (12) by a complex conjugate formula, averaging, and using the formula $\langle \widetilde{\varepsilon}(\mathbf{p}) \widetilde{\varepsilon}^*(\mathbf{p}') \rangle = \Phi_\varepsilon(\mathbf{p}) \delta(\mathbf{p}-\mathbf{p}')$ we obtain:

$$\langle \mathcal{E}\mathcal{E}^* \rangle = \frac{\pi^3 k^2 A_0^2 a^8 h^2}{8(R^2+k^2a^4)} \iiint_{-\infty}^{\infty} d^3p \Phi_\varepsilon(\mathbf{p}) \exp\left[-h^2(p_z-2k)^2 - \frac{1}{2}a^2(p_x^2+p_y^2)\right]. \tag{13}$$

We emphasize that (12) and (13) are correct both in the near zone of the antenna $(R \ll ka^2)$ and in the far zone $(R \gg ka^2)$. We can present formula (13) in the form

$$\langle \mathcal{E}\mathcal{E}^* \rangle = \frac{\pi^4 \sqrt{\pi} k^2 A_0^2 a^6 h}{4(R^2+k^2a^4)} \overline{\Phi}(0,0,2k), \tag{14}$$

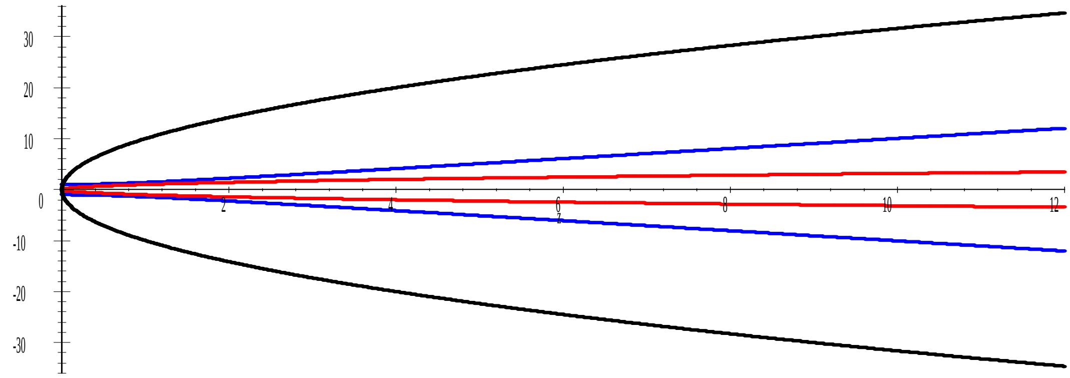

Fig. 2. Notations are the same as in Fig. 1. The value $\sqrt{\lambda R}/\gamma$ is shown in black.

where

$$\overline{\Phi}_\varepsilon\left(p_{0x}, p_{0y}, p_{0z}\right) \equiv$$
$$\iiint_{-\infty}^{\infty} d^3p\Phi_\varepsilon\left(\mathbf{p}\right) w\left(p_x - p_{0x}, p_y - p_{0y}, p_z - p_{0z}\right),$$
$$w\left(p_x - p_{0x}, p_y - p_{0y}, p_z - p_{0z}\right)$$
$$= \frac{ha^2}{2\pi\sqrt{\pi}} \exp\left\{-\frac{1}{2}a^2\left[\left(p_x - p_{0x}\right)^2 + \left(p_y - p_{0y}\right)^2\right] - h^2\left(p_z - p_{0z}\right)^2\right\}. \tag{15}$$

The function $w\left(p_x - p_{0x}, p_y - p_{0y}, p_z - p_{0z}\right)$ has a maximum value $ha^2/2\pi\sqrt{\pi}$ in the point $\mathbf{p}_0 = \left(p_{0x}, p_{0y}, p_{0z}\right)$, and satisfies the relation

$$\iiint_{-\infty}^{\infty} w\left(p_x - p_{0x}, p_y - p_{0y}, p_z - p_{0z}\right) d^3p = 1. \tag{16}$$

If $h \to \infty$, and $a \to \infty$ the function w tends to delta function. For finite values of h and a, w can be called a filtering (window) function having specific scales $1/a$ in x and y directions and a specific scale $1/h$ in the z direction. If the spectrum $\Phi_\varepsilon\left(\mathbf{p}\right)$ is a smooth function in the neighborhood of point $\left(0, 0, 2k\right)$, the value $\overline{\Phi}_\varepsilon\left(0, 0, 2k\right)$ is close to $\Phi_\varepsilon\left(0, 0, 2k\right)$, and formula (14) presents the scattered signal in terms of the Bragg component of the refractive index. In the opposite case, if $\Phi_\varepsilon\left(\mathbf{p}\right)$ varies significantly in the scales $1/a$ or $1/h$, formula (14) is also correct, but we cannot specify some isolated Bragg component of the spatial spectrum that determines the scattering.

3 Discussion

In conventional scattering theory (see, e.g. Tatarskii, 1967), the scattering intensity in the Fraunhofer approximation is presented in terms of the scattering cross section σ_0. Let $\mathbf{S}_{\text{sc}}\left(\mathbf{n}\right)$ be the energy flux of the scattered wave in the direction of the unit vector $\mathbf{n}$, and let $\mathbf{S}_0\left(\mathbf{n}_0\right)$ be the energy flux in the incident plane wave. Then, the scattering cross section σ_0 from the unit of scattering volume V to the unit of the solid angle, wrapping around the direction $\mathbf{n}$, is defined by the formula $\sigma_0\left(\mathbf{n}, \mathbf{n}_0\right) \equiv \mathbf{n}\mathbf{S}_{\text{sc}}\left(\mathbf{n}\right) R^2/V\left|\mathbf{S}_0\left(\mathbf{n}_0\right)\right|$. For scattering by dielectric permittivity fluctuations, the formula $\sigma_0 = 8\pi k^4 \sin^2\chi\widetilde{\Phi}_\varepsilon\left(k\mathbf{n} - k\mathbf{n}_0\right)$ was obtained in Tatarskii (1967) (see (4.19), page 68). Here, $\sin^2\chi$ describes polarization and is equal to 1 in our case, $\Phi_\varepsilon\left(\mathbf{p}\right)$ is the power spectrum of refractive index fluctuations, and $\widetilde{\Phi}_\varepsilon\left(\mathbf{p}\right)$ is the power spectrum, averaged in the $\mathbf{p}$-space over the domain of the order of value $8\pi^3/V$ (see Tatarskii, 1967, page 67).

In the case considered in this paper, we deal with the scattering of the Gaussian beam. The incident wave *does not have some fixed direction of incidence* and has a *different* energy flux in different parts of the scattering volume. In the case of incoherent summation of fields scattered from different parts of the scattering volume, it is possible to introduce the local values of scattering cross section and obtain the total cross section. However, the receiving antenna summarizes the scattered fields from different parts of the scattering volume *coherently*. In such conditions, we are unable to determine the scattering cross section using the common definition.

Instead of σ_0, we consider the ratio of $\langle\mathcal{E}\mathcal{E}^*\rangle$ to the analogous value $\left|\mathcal{E}_{\text{mirror}}\right|^2$, which is obtained from the reflection of the incident field from the plane mirror, which is positioned at the same distance as the center of the scattering slab. The value $\left|\mathcal{E}_{\text{mirror}}\right|^2$ may be easily calculated using the Fresnel approximation for the Green function and for $ka \gg 1$ the normalized dimensionless signal is given by the expression

$$\frac{\langle\mathcal{E}\mathcal{E}^*\rangle}{\left|\mathcal{E}_{\text{mirror}}\right|^2} = \pi^2\sqrt{\pi}\frac{k^2h}{a^2}\overline{\Phi}\left(0, 0, 2k\right). \tag{17}$$

In general, different approximations in a description of single scattering from random inhomogeneities differ only by the filtering function $w\left(\mathbf{p} - \mathbf{p}_0\right)$. In this aspect we must compare the Fresnel and the Fraunhofer approximations. Formulae for Fresnel scattering and for Fraunhofer scattering represent a scattered signal with different normalization, but nevertheless, we can compare them. In both cases the signal is proportional to the averaged Bragg component of the refractive index. The averaging operation (spectral window) in the case of Fresnel scattering is determined by the function

(15); the scales of averaging in $\mathbf{p}$-space (window width) are $1/a$ in the transverse to the beam directions and $1/h$ in the longitudinal direction. This means that only such details of spectrum shape, which have scales larger than $1/a$ and $1/h$, may be resolved. The ratio of the Bragg wave number $2k$ to the window transverse width $1/a$ can be called the transverse spatial quality factor Q_{tr}. For this quantity we have

$$Q_{\mathrm{tr}} = 2ka \sim \frac{1}{\gamma}. \tag{18}$$

As we mentioned in the Introduction, in the case of Fraunhofer scattering from statistically homogeneous fluctuations (distributed scatterers) we are unable to determine scattering volume consistently with scattering theory assumptions. Thus, the estimations for the window width (about $1/L_x, 1/L_y$, and $1/L_z$) obtained in the Fraunhofer approximation (see Tatarskii, 1967) can be questioned. Because the scales $L_x, L_y \sim \lambda R/a$ are much larger than a in the far zone of the antenna, the real width of the spectral window, $1/a$, is much wider and the corresponding quality factor is much less than in the Fraunhofer approach.

Another important difference between our and traditional consideration is related to the scattering volume V, entering in the formula $\sigma_0\left(\mathbf{n}, \mathbf{n}_0\right) \equiv \mathbf{n}\mathbf{S}_{\mathrm{sc}}\left(\mathbf{n}\right) R^2/V \left|\mathbf{S}_0\left(\mathbf{n}_0\right)\right|$. The value V for the real antenna remains uncertain (using different definitions of beam width we obtain V with different numerical coefficients). Because of this, if we use the intensity of radar signal to determine $\overline{\Phi}\left(0,0,2k\right)$, we introduce in $\overline{\Phi}$ some uncertain numerical factor. Formula (17) is free from this disadvantage.

4 Conclusion

Scattering of an electromagnetic field by refractive index fluctuations is considered using the Fresnel diffraction approximation. It is assumed that the EM pulse with a Gaussian temporal envelope is transmitted and received by a Gaussian antenna. Such consideration allows one to avoid usage of an uncertain scattering volume V and also eliminates the controversy between the definition of the scattering volume and the Fraunhofer zone scattering condition (see Introduction). The solution obtained is correct both in the far and the near zones of the antenna. There is no need to assume any relations between the correlation scale (outer scale) L_0 of inhomogeneities, the radius of the first Fresnel zone, and the size of the scattering volume, as was done in Tatarskii (1971). There is no need to assume an incoherent summation of fields that are scattered by different parts of the scattering volume; if such incoherent summation takes place, it would be a particular case of the general formula (17). The window shape obtained in the Fresnel approximation is given by the formula (15) and is determined by the scales of antenna a and duration of pulse h.

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