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Investigation on the weather radar equation at attenuating wavelength

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Abstract. The *classical* weather radar equation is here generalized to include a range-bin extinction effect due to rainfall path attenuation within each range bin. It is shown that only in the case of low-to-moderate attenuating media, the derived range-bin extinction factor is, by definition, closed to one so that the *classical* radar equation can be used. These theoretical results are also obtained by using a microwave radiative transfer approach. Within the assumption of first-order scattering, a new definition of the radar reflectivity (in terms of backscattered specific intensity) yields the same generalized radar equation. Numerical results confirm that the effect of the range-bin extinction factor, depending on frequency and range resolution, can be significant at X band for intense rain, while at Ka band and above even for moderate rain.

1 Introduction

Microwave weather (or meteorological) radars represent a well established technique to retrieve rainfall structure and microphysics (e.g., Bringi and Chandrasekar, 2001). Due to constraints on the component sizes (e.g., waveguides, antenna reflector) and high power transmitter requested at S band, higher radar frequencies have been considered for operational purposes (Sauvageot, 1992). As a matter of fact, for ground-based systems C and X bands have been also selected, while for space-based sensors Ku to W bands have been taken into consideration for cloud and rain retrieval (e.g., Delrieu et al., 2000; Meneghini et al., 1983).

When operating a radar at attenuating wavelength, path attenuation needs to be included in the equation governing the quantitative analysis of radar measurements (Meneghini, 1978; Sauvageot, 1992). As known, hydrometeor path attenuation increases as the frequency increases beyond S band (Sauvageot, 1992). Any radar technique above S band must take into account, and possibly remove, path attenuation effects in order to correctly convert measured reflectivity into rain rate. To this aim, iterative and constrained methods have been proposed to process radar data either from ground-based or from space-based systems (e.g., Meneghini et al., 1983; Marzano et al., 1999; Testud et al., 2000).

Neverthless, the validity of the *classical* radar equation in attenuating media is not clearly stated in literature. For frequencies higher than S band and depending on rainfall intensity, the increase of path attenuation is expected to correspond to the increase of the volumetric albedo and the scattering asymmetry of hydrometeors (Ishimaru, 1978; Marzano et al., 2000; de Wolf et al., 2000). Limitations to the use of the *classical* radar equation in certain observational circumstances can be obtained only if the radar equation itself is framed within a more general theory which includes multiple scattering phenomena.

In this work, the *classical* radar equation (for simplicity, considered for a single-polarization or for unpolarized radiation) is generalized to include such a range-bin extinction effect. Numerical simulations are performed by using statistical relations between reflectivity, rainrate and specific attenuation taken from experimental results available in literature.

2 Extended radar equation at attenuating wavelength

The equation of a pulsed meteorological radar, operating in an attenuating medium, basically relates the mean received power $< P_R(r, \Omega_0) >$, obtained from averaging radar echoes due the scattering volume bin ΔV_r at a range r in the pointing direction Ω_0, to the transmitted power P_T.

Let us suppose an observation geometry, where the radar is placed in the origin. For simplicity of notation, besides the spherical coordinates (r, θ, ϕ) with respect to the z axis, we introduce a slant reference coordinate system (r', θ_r, ϕ_r) with the corresponding solid angle $\Omega_r = (\theta_r, \phi_r)$. In presence of an inhomogeneous attenuating medium, the radar

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equation can be stated as follows (Savaugeot, 1991):

$$< P_R(r,\Omega_0) >= \frac{P_T \lambda^2 L^2(r)}{(4\pi)^3} \int_{\Delta V_r} \eta(r',\Omega_r) \frac{G^2(\Omega_r)}{r'^2} e^{-2\tau_r(r')} d^3 r' \quad (1)$$

where λ is the radar wavelength (in vacuum), ΔV_r is the radar resolution volume or cell (spanned by coordinates r', θ_r and ϕ_r), L is the one-way path attenuation factor from the radar antenna to the considered range bin, η is the volumetric radar reflectivity, τ_r is the optical thickness (or path attenuation A) along the range r, d^3r' is the elementary volume within the radar bin. The antenna gain function $G(\Omega_r)$ along Ω_0 is such that $G(\Omega_r) = G_0 \mid f_n(\Omega_r) \mid^2$, with $G_0 = G(\Omega_0)$ the maximum gain in the pointing direction Ω_0 and $\mid f_n(\Omega_r) \mid^2$ the one-way normalized radiation pattern.

The volumetric radar reflectivity η in (1) can be related to the *equivalent reflectivity factor* Z_e by the well-known relation:

$$\eta(r,\Omega_r) = \frac{\pi^5 \mid K \mid^2}{\lambda^4} Z_e(r,\Omega_r) \quad (2)$$

where K is the medium polarizability complex factor, while the path attenuation factor can be expressed as:

$$L(r,\Omega_r) \cong L(r,\Omega_0) \equiv L(r) = e^{-\tau(r)} = e^{-\int_0^r k(r')dr'} \quad (3)$$

being k the volumetric specific attenuation (or extinction coefficient) and having supposed a weak dependence of L on the observation angle Ω_r within the beam.

If the integration in (1) is extended to the finite range bin Δ_r (i.e., range resolution) and to the antenna main-lobe beamwidth Ω_M, (defined as either the -3dB or the -6dB solid angle), (1) can be easily re-expressed. Moreover, if the radar antenna is sufficiently directive such that the scattering volume ΔV_r can be assumed to be uniformly filled by randomly-distributed scatterers, the equivalent reflectivity factor Z_e and specific attenuation k become independent of the observation angle Ω_r and range r' within the radar volume. In these circumstances (1) can be simplified by introducing the antenna two-way radiation solid angle Ω_{2A} (Marzano et al., 2000). If the antenna power pattern is assumed to be Gaussian, the expression of Ω_{2A} yields the well-known Probert-Jones correction factor to the radar equation (Sauvageot, 1992).

Under these approximations, the last integral in r' in (1) can be easily calculated by performing an integration by parts. Assuming $\Delta r \ll r$, that is for large distances from the radar with respect to the range resolution, the mean received power from an arbitrary range bin can be rearranged as follows:

$$< P_R(r,\Omega_0) >\cong \frac{P_T \pi^2 \mid K \mid^2 G_0^2 \Omega_{2A} \Delta_r}{64\lambda^2} \frac{Z_e(r,\Omega_0) f_b(r)}{r^2} L^2(r) \quad (4)$$

where $f_b(r)$ is the *range-bin extinction factor*, defined by:

$$f_b(r) \equiv \frac{1}{2k\Delta_r}\left(1-e^{-2k\Delta r}\right) = \frac{1}{2\Delta\tau_r(r)}\left(1-e^{-2\Delta\tau_r(r)}\right) \quad (5)$$

with $\Delta\tau_r$ the range-bin optical thickness. Equation (5) expresses the effect of specific attenuation and/or range resolution within the considered radar scattering volume.

A comparison of (4) with the *classical* radar equation suggests to define a *bin-averaged equivalent reflectivity factor* $Z_{eb}(r,\,\Omega_0)$ at a range r as:

$$Z_{eb}(r,\Omega_0) \equiv Z_e(r,\Omega_0) f_b(r) \quad (6)$$

so that the mean received power can be expressed by the following *extended* radar equation:

$$< P_R(r,\Omega_0) >\cong C \frac{Z_{eb}(r,\Omega_0)}{r^2} L^2(r) \quad (7)$$

where C is the *radar instrumental constant*.

It is worth mentioning that, if either the specific attenuation k is small or the range resolution Δr is short or better the range-bin optical thickness $\Delta\tau_r$ is much less than 1, then the range-bin extinction factor in (5) reduces to 1. Under these conditions, (7) becomes identical to the *classical* radar equation in attenuating media. We can conclude that the latter is a particular case of the *extended* radar equation (7).

3 Radiative transfer theory

Scattering and propagation characteristics of radiowaves through a random medium can be profitably studied by using the *radiative transfer theory* (RTT). The need to consider higher operating frequencies (e.g., for technological reasons spaceborne radars cannot support large antennas) has recently raised the issue of quantifying incoherent backscattered radiation in radar meteorology problems (Marzano et al., 2000; de Wolf et al., 2000).

The fundamental quantity in the radiative transfer theory is the *specific intensity I*, also called *radiance*, measured in [W m^{-2} sr^{-1} Hz^{-1}] (Ishimaru, 1978). Under the assumption of unpolarized radiation, the specific intensity of the scattered radiation is solution of a scalar differential-integral equation, known as radiative transfer equation (RTE).

3.1 Generalized radar equation

The *apparent radar reflectivity* η_a, defined as the ensemble average of backscattering cross sections of all the particles within a unit volume, can be related to backscattered specific intensity $I_R(r,\,\Omega_r)$ by means of the relationship (Marzano et al., 2000):

$$\eta_a(r,\Omega_r) = \frac{4\pi}{\Delta_r} \frac{< I_R(r,\Omega_r) >}{F_T(r,-\Omega_r)} \quad (8)$$

where F_T is the transmitted power flux density at range r. In analogy to (2), the *apparent reflectivity* η_a can be expressed through the *apparent equivalent reflectivity* factor:

$$Z_a(r,\Omega_r) \equiv \frac{\lambda^4}{\pi_5|K|^2}\left[\frac{4\pi}{\Delta_r}\frac{< I_R(r,\Omega_r) >}{F_T(r,\Omega_r)}\right] \tag{9}$$

Given the mean value $< I_R(r,\Omega_r) >$ of the *apparent* received specific intensity, the *apparent* backscattered received power $< P_{Ra}(r,\Omega_0) >$ can be expressed as (Ishimaru, 1978):

$$< P_{Ra}(r,\Omega_0) >= \frac{\lambda^2}{4\pi}\int_{4\pi} G(\Omega_r) < I_R(r,\Omega_r) > d\Omega_r \tag{10}$$

where the directive gain G has been related to the antenna equivalent area A_e through the reciprocity formula, being $G_0 = (4\pi/\lambda^2)A_{e0}$.

If $< I_R(r,\Omega_r) >$, and thus $\eta_a(r,\Omega_r)$, can be assumed constant within the radar scattering volume, it is straightforward to re-express the mean apparent received power. Thus, substituting (8) and (9) into (10), we obtain the *generalized radar equation* which resembles the *classical* radar equation:

$$< P_{Ra}(r,\Omega_0) >\cong C\frac{Z_a(r,\Omega_0)}{r^2} \tag{11}$$

where C is again the radar constant. Notice that the *generalized radar equation* in (11) can, in principle, take into account multiple scattering effects (Marzano et al., 2000).

3.2 Apparent and bin-averaged equivalent reflectivity

The aim of this section is to derived an explicit expression of Z_a, given in (9), from RTT. We limit our analysis to the first-order scattering following an iterative solution method. Let us consider a simple atmospheric model consisting of a finite homogeneous atmospheric slab whose bounds are $\tau = 0$ (i.e., $z = 0$) and $\tau = \tau_r$. If the specific attenuation k is constant, then the geometrical thickness of the slab is $\Delta r = \tau_r/k$. As in Sect. 2, the radar antenna is placed in $\tau = 0$ with a vertical pointing direction along z, being $L(r) = 0$ in this example.

Under the above mentioned assumptions and by applying proper boundary conditions, the diffuse specific intensity $I^{(1)}(0,-\mu,\phi)$ in the first-order scattering (FOS) approximation of the RTE solution is given by (Ishimaru, 1978):

$$I^{(1)}(0,\Omega_0) =< I_R(r,\Omega_0) >_{FOS}= \frac{wF_0}{8\pi}p(-\Omega_0,\Omega_0)\left(1-e^{-2\tau_r}\right) \tag{12}$$

where $p(\Omega,\Omega_0)$ is the phase function and $w = k_s/k$ is the single scattering albedo with k_s the volumetric scattering coefficient.

The above expression, substituted in (8), allows one to calculate the apparent radar reflectivity with slant depth Δr:

$$\eta_{aFOS}(r,\Omega_0) = \frac{4\pi}{\Delta r}\frac{< I_R(r,\Omega_o) >_{FOS}}{F_0} = \frac{k_s p(-\Omega_0,\Omega_0)}{2k\Delta r}\left(1-e^{-2\tau_r}\right) \tag{13}$$

noting that $F_T(r,\Omega_0) = F_0$. Previous equation can be rewritten as:

$$Z_{aFOS}(r,\Omega_0) = Z_e(r,\Omega_0)\frac{\left(1-e^{-2\Delta\tau_r}\right)}{2\Delta\tau_r} \tag{14}$$

being $\Delta\tau_r = \tau_r$ for the consider slab geometry and, by definition, the radar reflectivity $\eta = k_s p$.

In (14) we recognize the range-bin extinction factor $f_b(r)$, defined in (5). Thus, an identical expression is found for both the apparent equivalent reflectivity factor Z_{aFOS} and the bin-averaged equivalent reflectivity factor Z_{eb}, given in (6):

$$Z_{aFOS}(r,\Omega_0) = Z_e(r,\Omega_0)f_b(r) = Z_{eb}(r,\Omega_0) \tag{15}$$

Previous results have been obtained for a single homogeneous slab. The expression of backscattered radiance can be generalized in a straightforward manner to a relationship valid for the i-th range bin of a layered atmosphere. Thus, Z_{aFOS} should include path attenuation factor L as well.

4 Numerical results

The relation between equivalent reflectivity factor Z_e, specific attenuation k and rainfall rate R are well documented in literature and are generally assumed to have a power-law forms (Sauvageot, 1992):

$$Z_e = aR^b \ \ \& \ \ k = cR^d \tag{16}$$

where a, b, c and d are regression coefficients. The latter are usually derived from the statistical analysis of *in situ* data, coming from rain gauges, disdrometers ans aircraft probes (e.g., Sauvageot, 1992; Bringi and Chandrasekar, 2001). We have considered here various operational frequencies to evaluate the range-bin extinction factor. As previously mentioned, ground-based applications are generally focused on the use of C, X and Ka band, while airborne and spaceborne radars have been deployed at Ku, K, Ka and W band. In this respect, it is worth recalling that the Precipitation Radar (PR) at Ku band has been the first rain radar successfully launched in space aboard the Tropical Rainfall Mission (TRMM) platform.

From open literature, we have taken $Z_e - R$ and $k - R$ relative to orographic thunderstorms at C and X band derived by Delrieu et al., 2000 (i.e., their Table 2 at 10° C). At Ku, K, Ka and W band we have resorted to tropical rainfall scenarios by using $Z_e - R$ and $k - R$ from Haddad et al., 1997 (their Tables IX-XII for $D" = 1.0$). Fig. 1 depicts the behavior of these power-law relations for 4 frequency bands (i.e., X, Ku, K and Ka band) as a function of rainrate R.

From its definition, f_b is dependent on the optical thickness (or attenuation) $\Delta\tau_r$ of the range bin. Thus, for a given R, being k determined by curves in Fig. 1, we need to specify the range bin resolution Δr. The latter, in accordance with common operational requirements, has been set to 125, 250, 500 and 1000 m, respectively.

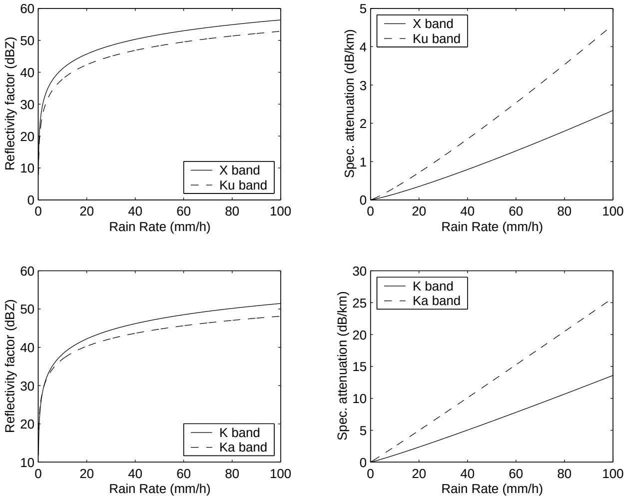

Fig. 1. Statistical relations $Z_e - R$ and $k - R$ derived from Delrieu et al., 2000 X band and from Haddad et al., 1997 at Ku, K, and Ka band.

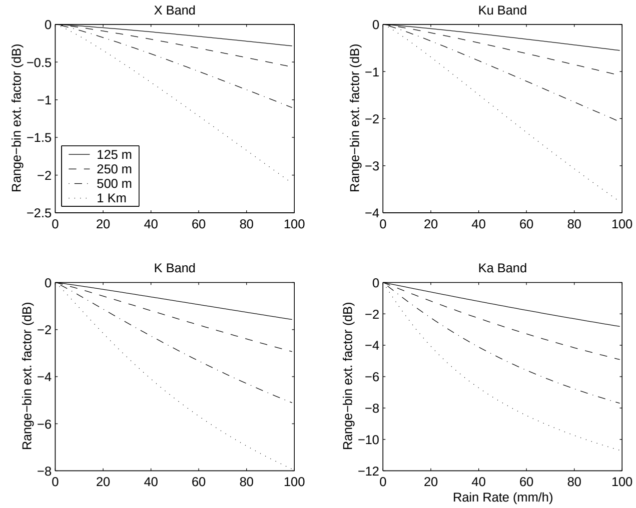

Fig. 2. Range-bin extinction factor as a function of rainrate, derived using relationships plotted in Fig. 1 for the same frequency bands and as a function of the range-bin resolution.

Figure 2 illustrates the impact of the range-bin extinction factor f_b on the computation of the apparent radar reflectivity Z_a for the same frequency bands of Fig. 1, having Δr as a parameter. We remind that, in case of negligible impact, it should $f_b = 1$ (or $f_b = 0\,\mathrm{dB}$).

As expected, for a given rainrate, the range-bin extinction factor f_b is less than 1 (or less than 0 dB) for increasing frequency and range resolution. While at C band, $f_b > -0.5\,\mathrm{dB}$ up to 100 mm/h (not shown), at K_a band $f_b < -1\,\mathrm{dB}$ for $R > 20\,\mathrm{mm/h}$ and $\Delta r < 500\,\mathrm{m}$.

As a final consideration, let us suppose to operate at X band observing a rain slab with 20 mm/h, characterized by Z_{e0}. For $\Delta r = 125\,\mathrm{m}$, $f_b = 0.99$, while for $\Delta r = 1\,\mathrm{km}$ f_b is about 0.92. This means that, if the measured reflectivity is averaged using N bins, for its spatial average it holds:

$$\overline{Z}_{eb} = Z_{e0} f_b(r,;\, N\Delta r) \neq Z_{e0} \frac{\sum_{i=1}^{N} f_b^{(i)}(r;\, \Delta r)}{N} \qquad (17)$$

Since it is operationally fairly common to provide products at a degraded range resolution, last inequality should be kept in mind when processing radar data in attenuating media.

5 Summary and conclusions

The *classical* radar equation has been here generalized to include with a range-bin extinction effect. It has been shown that only in the case of low-to-moderate attenuating media, the derived range-bin extinction factor is, by definition, closed to one so that the *classical* radar equation can be used. This theoretical analysis has supported the conclusion that radar analysis in strong attenuating media should include first-order scattering effects. Preliminary numerical simulations have been also performed. Results confirm that the effect of the range- bin extinction factor, depending on frequency and range resolution, can be relevant at X band for intense rain and at Ka band and above even for moderate rain.

An impact in operational procedures of reflectivity spatial averaging is expected when significant attenuation due to rainfall is observed. Finally, the discrepancy due to this range-bin extinction factor, if not properly taken into account, can translate into relevant systematic errors of estimated rainrates and instabilities of path-attenuation correction algorithms.

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