Proceedings of ERAD (2002): 11–16 © Copernicus GmbH 2002

ERAD 2002

A variational method for attenuation correction of radar signal

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Abstract. Most operational radars in Europe, Canada and Asia are C-band radars. Therefore, they are significantly affected by wet radome and precipitation attenuation, which makes quantitative use of radars difficult.

Most of the proposed attenuation correction methodologies use different sources of information (ground-echo measurements, dual-radar systems ...) to constrain the correction. Here, using the equation of Hitschfeld and Bordan (1954) and rain gage data as constraints, a correction based on the minimization of a cost function is suggested. In this cost function, control variables are the corrected rainfall field, a "recalibration" constant of the radar and the parameters of the DSD. The method can incorporate other sources of information to further constrain the problem.

Here, errors due to C-band attenuation are analyzed by simulating C-band data from non-attenuated S-band data. The simulated C-band data are then compared with the S-band data to derive long-term statistics and the Path Integrated Attenuation (PIA) is compared with the Path Integrated Reflectivity. Wet radome attenuation is estimated from the variation in the spatial average reflectivity over a large area (assuming that it is more important than the natural variability of precipitation fields).

1 Introduction

Traditionally, the choice of the operating wavelength of a weather radar has been faced up as a compromise between the high cost of almost non-attenuated S-band and dealing with moderate attenuation rates of C-band radars.

This is the reason why many operational radars in Europe, Canada and Asia (both for meteorological and hydrological uses) are C-band radars and, therefore, attenuation is an obstacle for quantitative use. To illustrate this, Duncan et al. (1991) found that the signal attenuation by rain produced 20% underestimation of accumulated rainfall and a maximum value of area affected by total attenuation of 5% in a study over 6 events. In a similar way, Delrieu et al. (2000) studied attenuation statistics at different ranges on the south of France, obtaining more than 5% of rainrate profiles exceeding a Path Integrated Attenuation of 3 dB at 50 km.

Historically, the studies by Atlas and Banks (1951) and Hitschfeld and Bordan (1954) induced the progressive abandonment of most attenuated wavelengths ($\lambda <5$ cm) because of their high degree of signal attenuation and the difficulty of correction. In 1954, Hitschfeld and Bordan derived the analytical solution of the attenuation correction supposing a potential Z-k relation. However, they also stated the high instability of this expression (particularly with small errors in the radar calibration constant) that makes the correction useless "unless the calibration error can be kept within extremely narrow limits". They already proposed the use of "a gage deep in the storm" to reduce the radar calibration error.

However, the use of attenuated-frequencies for air or satellite-borne radars promoted the development of different methods to correct for attenuation (using single or double-frequency, single or double-beam and single or double-polarized radars). Most of the single-frequency methodologies (Meneghini et al., 1983, Iguchi and Meneghini, 1994, Marzoug and Amayenc, 1994, Iguchi et al., 2000) are based on the Surface Reference Technique (SRT) that consists on constraining the Path Integrated Attenuation with the surface return. In a similar way, Delrieu et al. (1997) proposed the use of mountain returns to correct for attenuation of ground-based radars.

Here, a methodology to correct for attenuation (mainly in C-band radars) is presented. It consists on the minimization of a cost function, that includes the equation derived by Hitschfeld and Bordan (1954) and as many constraints as available. In a preliminary form, it uses punctual measurements of rainfall as constraints.

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2 Attenuation theory/attenuation equations

From the radar equation (see e.g. Doviak and Zrnic, 1992), the measured (attenuated) reflectivity at a range r can be written as:

$$Z_m(r) = \frac{P_r(r)\cdot r^2}{C} \tag{1}$$

where $P_r(r)$ is the received power and C is the radar constant that depends on radar paramaters such as peak power, wavelength, antenna gain, beam width, pulse duration, ...

The measured reflectivity can be expressed as:

$$\begin{aligned} Z_m(r) &= Z(r)\cdot A(r) \\ &= Z(r)\cdot \exp\left(-0.2\cdot\ln(10)\cdot\int_0^r k(s)ds\right) \end{aligned} \tag{2}$$

where,

$Z(r)$: "equivalent" (non-attenuated) reflectivity factor (mm^6 m^{-3}).

$A(r)$: path integrated attenuation factor at a range r.

$k(s)$: specific attenuation (dB km^{-1}).

Assuming that the specific attenuation can be related to $Z(r)$ by a power-law,

$$k(r) = \alpha\cdot Z(r)^\beta \tag{3}$$

Hitschfeld and Bordan (1954) expressed (2) as (see the development of the current expression in Meneghini, 1978):

$$Z(r) = \frac{Z_m(r)}{\left[1-0.2\cdot\ln(10)\cdot\beta\int_0^r \alpha\cdot Z_m(s)^\beta\, ds\right]^{\frac{1}{\beta}}} \tag{4}$$

In principle, Eq. (4) is analytically obtained from (2), but small errors in the denominator (errors in Z_m, in the parameters, ...) may cause the divergence of the solution. Because of this, it makes necessary to constrain the solution with independent measurements of path-integrated-attenuation (estimated, for example, from ground echoes).

3 Methodology

The presented methodology is conceptually very simple: it combines information given by the analytical solution (4) and measurements able to be used as constraints.

It consists on the minimization of a cost function (5). The first part of the cost function (first summand) uses the corrected (non-attenuated) rainfall field as background. The second part (the rest) represents the observation term, constraining the result with observations from different sources. In a first formulation, the cost function uses punctual measurements of rainfall rate (for instance, given by rain gages):

$$\begin{aligned} J(\Delta c, a, b, \alpha, \beta, R) &= w_1\cdot\sum_{j=1}^{n_{bins}}\left(R_i - R_i^{H-B^*}\right)^2 \\ &+ w_2\cdot\sum_{j=1}^{n_g}\left(R_j - R_{gj}\right)^2 + w_3\cdot\sum_{j=1}^{n_g}\left(R_j^{H-B^*} - R_{gj}\right)^2 \end{aligned} \tag{5}$$

where,

a, b: parameters of a power-law Z-R relationship.

Δc: estimation of the radar calibration error (in dB).

w_i: weights associated to each term of the cost function.

n_{bins}: number of bins where the reflectivity field is measured by the radar.

R_i: rainfall rate at the bin i ($i = 1, \ldots, n_{bins}$). This is the field to be derived from the minimization of the cost function.

n_g: number of gages.

$R_{g,j}$: rainfall rate measured at the gage j ($j = 1, \ldots, n_g$).

$R_i^{H-B^*}$: rainfall rate estimated from the measurement of the reflectivity field after being corrected using the Hitschfeld and Bordan algorithm.

$$R_i^{H-B^*} = \left(\frac{Z_i^{H-B^*}}{a}\right)^{\frac{1}{b}} \tag{6}$$

$Z_i^{H-B^*}$ is the non-attenuated reflectivity estimated using the Hitschfeld and Bordan expression, but taking into account that the measured reflectivity may be affected by an error in the radar constant (Δc):

$$Z_j^{H-B^*} = \frac{Z_m(r_j)}{\left[10^{\frac{\beta}{10}\Delta c} - 0.2\cdot\ln(10)\cdot\beta\int_0^{r_j}\alpha\cdot Z_m^\beta(s)\, ds\right]^{\frac{1}{\beta}}} \tag{7}$$

The first term in the cost function (5), accounts for the similarity of the derived rainfall field and the solution of the Hitschfeld and Bordan equation when it is applied to correct the measurement of radar affected by a calibration error (7). The second term constrains the rainfall with the measurements in the rain gages, while the third summand is redundant, useful to give more robustness to the solution.

It is important to state that the correction does not impose the control variables to take a given value (for example, the derived rainfall field in the gages should not take the gage measurement), because it supposes that the different terms may be affected by some uncertainties. However, the confidence in each term may be controlled with the weights w_i given to each term in the cost function. Another advantage of this methodology is that additional constraints may be introduced very easily in the cost function.

4 Application

In order to assess the behavior of the proposed methodology, it has been applied in a simulation context using the first elevation of an S-band radar, under the hypothesis of knowing the Z-R and Z-k relationships. Because of this, the control variables to be derived from the proposed methodology are the calibration error (Δc) and the rainfall field (R). The application of the methodology has been done correcting each

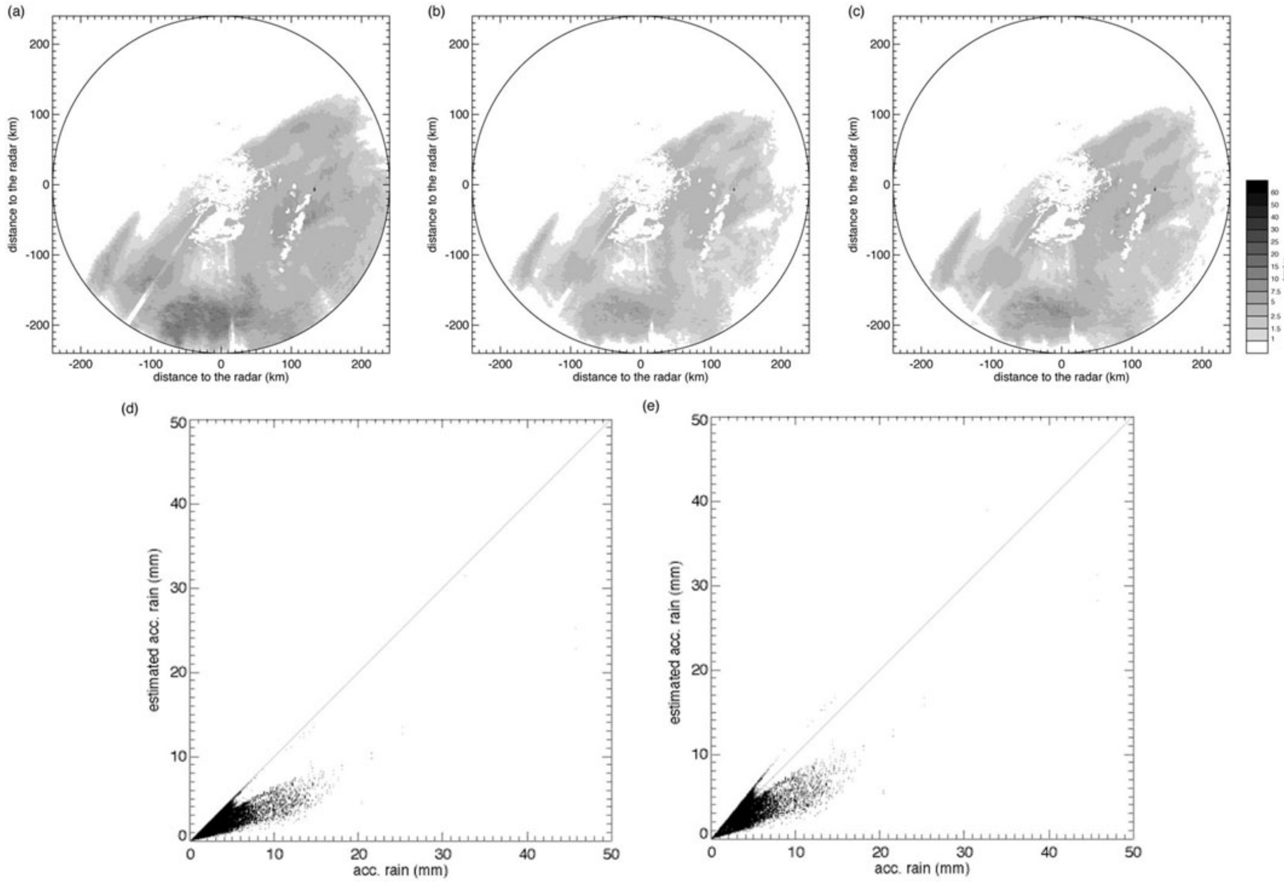

Fig. 1. Simulation of the rainfall field measured by a C-band radar affected by a calibration error of +1.5 dB. **(a)** Accumulated rainfall field estimated from the measurement of McGill S-band radar (the reference); **(b)** accumulated rainfall field estimated from the simulated measurement of a C-band radar; **(c)** accumulated rainfall field estimated from the simulated measurement of a C-band radar affected by a calibration error of +1.5 dB. In the lower part, scatter plots of the attenuated rainfall accumulations versus the reference field when the radar is well-calibrated **(d)** and when it is affected by a bias of +1.5 dB (e).

scan separately, estimating the calibration error every 5 minutes.

The simulation has been done over a 1-hour dataset registered by McGill S-band Doppler radar on 16 September 1999. The only correction that has been applied to these data is the identification and elimination of ground echoes. Therefore, it is easy to see in the accumulation map several gaps corresponding to ground echoes or some azimuths affected by partial beam blocking (see Fig. 1a). However, this S-band dataset has been used to simulate both the C-band (attenuated) reflectivity field and rain gage measurements and to estimate a "reference" rainfall field (applying the Z-R relationship of Marshall and Palmer, 1948), supposing that it is perfectly calibrated and that it does not suffer from attenuation, to compare the derived rainfall field with.

4.1 Simulation

Both C-band (attenuated) reflectivity fields and rain gages measurements have been simulated. S-band reflectivity fields have been attenuated using a given Z-k relationship applying the attenuation equation (2) to simulate attenuated C-band measurements. A power-law Z-k relationship has been considered (3) using the coefficients proposed by Battan (1973) for C-band wavelengths ($\alpha = 2.24 \cdot 10^{-4}$; $\beta = 0.62$).

After the simulation of the attenuated data, a bias of +1.5 dB has been introduced in the radar system to simulate a calibration error (see Fig. 1), which is supposed to be corrected by the proposed methodology. The rain rate measurements made by a uniformly distributed network of rain gages are also simulated (see Fig. 2a). To do this, an unbiased random error (uniformly distributed in the interval (-100%,+100%)) is introduced in the estimates of rainfall from the S-band over the gage locations to simulate the discrepancies between radar and rain gages measurements, normally observed in reality (Fig. 2).

4.2 Results

In this section the results obtained after the application of the proposed methodology are presented. Figure 3 shows a comparison between the estimates of 1-hour accumulations obtained from the attenuated C-band measurements and the corrected rainfall field. The results show a quite good im-

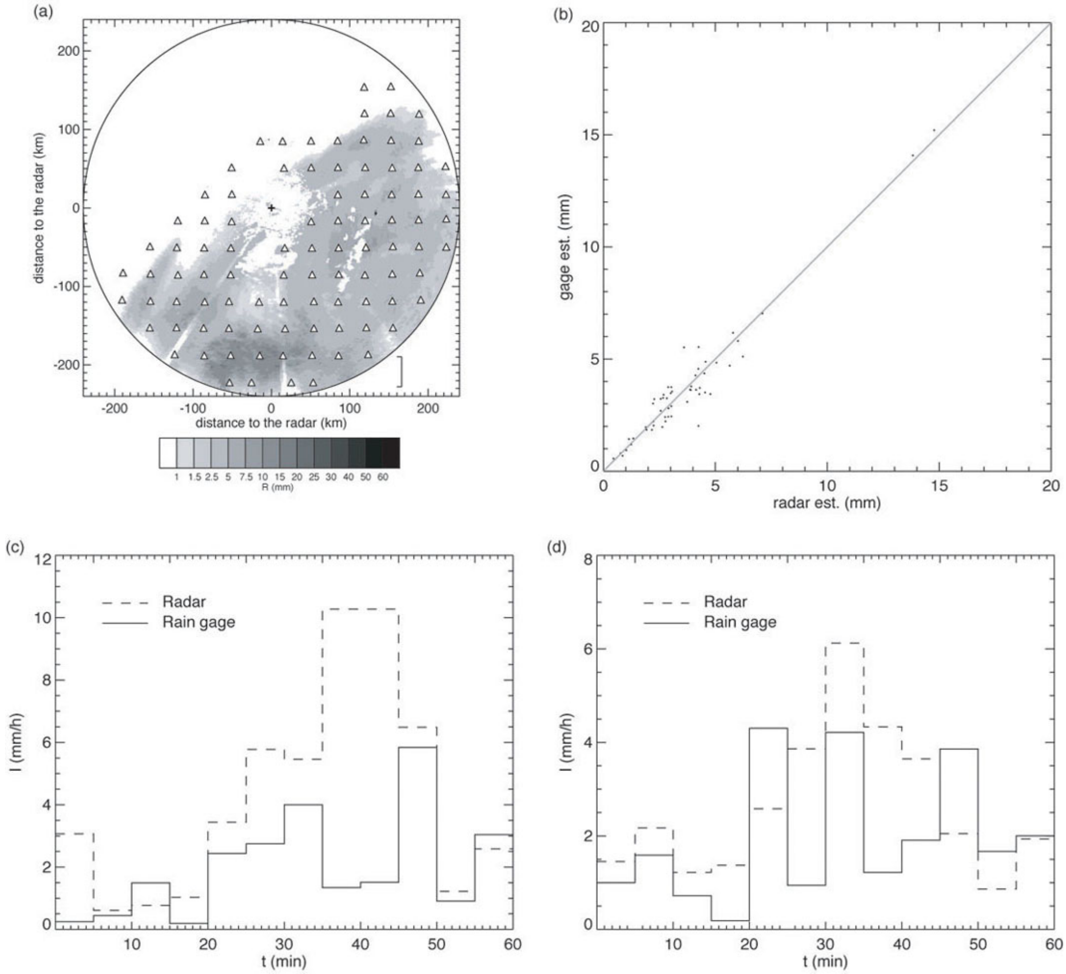

Fig. 2. Simulation of rainfall measured by a network of rain gages **(a)**, over the map of 1-hour accumulated rainfall. **(b)** Scatter plot of the 1-hour accumulated rainfall simulated at the gage points versus the S-band radar estimate. **(c)** and **(d)** Comparison of the S-band radar estimate of rainfall at two rain gage points (dotted line) and the simulation of the measurements at the gages (continuous line).

provement in the 1-hour accumulated rainfall field with respect to the estimate obtained from an attenuated radar (correcting the bias and reducing the scattering).

The proposed methodology has fixed the calibration error. In spite of the discrepancies introduced between the gage measurements of rainfall and the reference at these points, the applied methodology seems to be little affected. However, it is important to point out that when there is a bias in the measurements at the gages, it is reproduced by the estimated calibration error that introduces this tendency in the derived rainfall field. To avoid it and to increase the robustness of the method, it is important to introduce additional constraints in future developments. Another way to solve this may be to consider the calibration error to be nearly constant and able to be estimated working with more than one radar scan.

On the other hand, the hypothesis of knowing the parameters of the Z-R and Z-k relationships seems to be too hard, and this may be a reason of the good results obtained.

5 Conclusions

Attenuation is an important difficulty for quantitative use of C-band radars, for which it is not negligible. However, operative C-band radars do not use to be corrected because of the instability of the correction equations with different sources of errors.

A variational method based on the minimization of a cost function is proposed for the correction of the attenuation effects in C-band radars using (in preliminary development) punctual measurements of rainfall to constrain the correction equation in which the radar calibration error is introduced. The form of the cost function avoids the divergence of the analytical solution and allows more constraints to be easily incorporated.

The results obtained in a simulation case under important simplifications seem to be encouraging and more development should be made. The comparison of the derived 1-hour accumulated rainfall has shown that the proposed methodology has removed part of the scattering and the bias observed

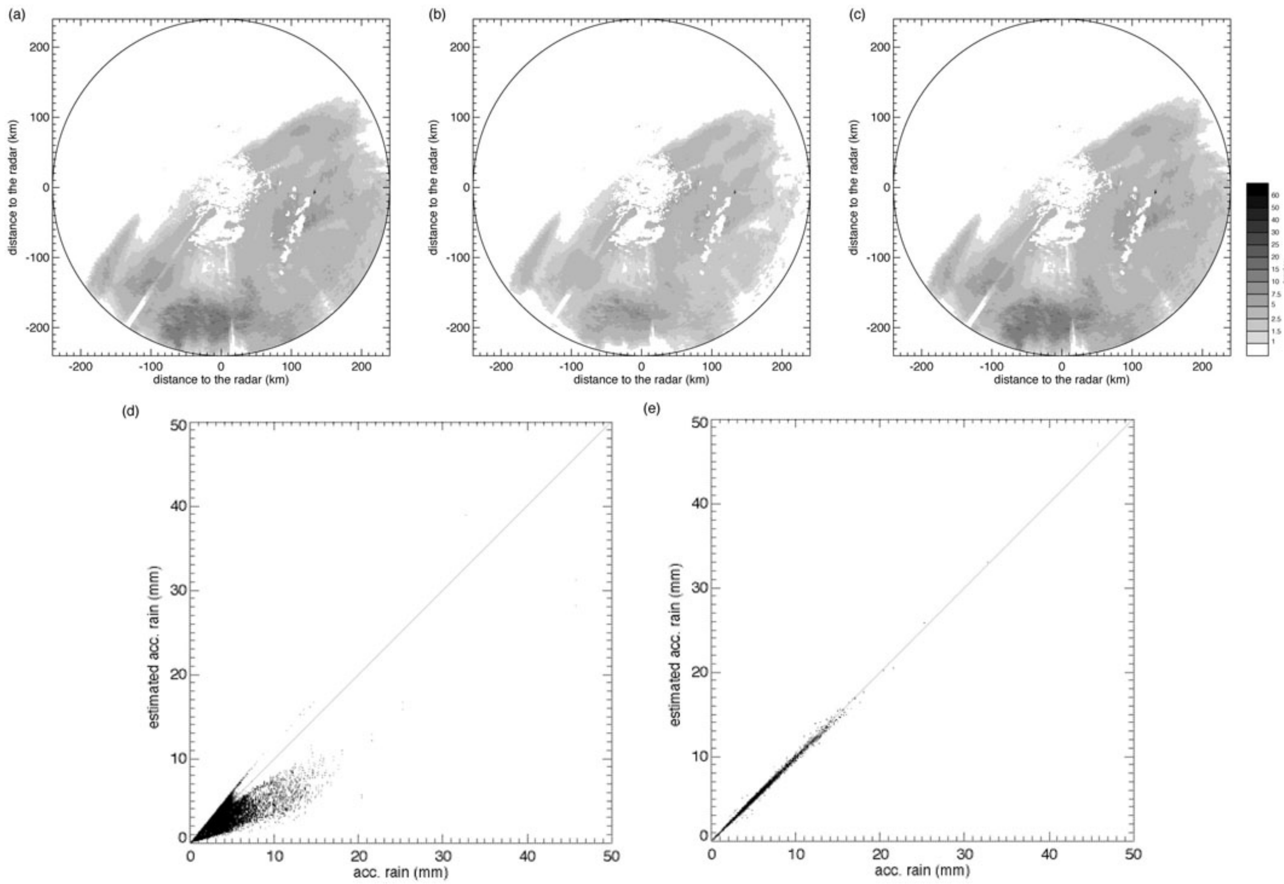

Fig. 3. Comparison between the estimates of 1-hour accumulated rainfall field made from: **(a)** the reflectivity field measured by the McGill S-band radar; **(b)** the simulated reflectivity field measured by a C-band radar affected by a calibration error of +1.5 dB; **(c)** the corrected rainfall field. **(d)** Scatter plot of the estimation of accumulated rainfall made from the simulated C-band measurement versus the reference; **(e)** scatter plot of the corrected accumulated rainfall field versus the reference.

in the estimation of rainfall from a C-band radar. However, uncertainties in the DSD parameters must have an important effect, introducing important scatter in the results. Therefore, this is a key point to be assessed in future work.

Although we have used rain gages to constrain the correction, this does not seem to be a perfect solution and more constraints should be used. An interesting point to take into account is the need of constraining the correction using radar measurements. Among the additional constraints able to be used we will consider:

– The attenuation constraint estimated from mountain returns (similarly to what was proposed by Delrieu et al., 1997).

– An additional constraint may be introduced if the measurements of two ore more radars were available in an overlapping region, taking the idea of Srivastava and Tian (1996).

Acknowledgement. This work has been carried out in the framework of the CICYT project REN2000-1755-C03-01, and of the EHIMI project, funded by the Agència Catalana de l'Aigua.

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