

The role of particular rain drop size classes on specific rain attenuation at various frequencies with Czech data example

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Abstract. The influence of the DSD on rain attenuation computation is studied in the 10–150 GHz frequency region. The influence of particular rain drop sizes (diameters) is shown concluding that the prevailing contribution is caused by drops of the diameter close to 0.7–1.5 mm when the importance of small rain drops is increasing with the increasing frequency. A 2 step model is suggested not to be able to derive the attenuation only but also to model the rain drop spectra accurately. Conclusions are derived from the typical DSD (which was measured in the Czech Republic and corresponds to the 2.17 mm/h rain rate). The convenience of DSD analytical approximations (DSD models) is also discussed.

1 Introduction

Rain is one of the most important atmospheric phenomena influencing the attenuation of microwaves. The rain drop size distribution (DSD) and its appropriate parameterisation plays an important role in the proper interpretation of the meteorological radar measurement as well as in the estimation of the radiowave attenuation due to rain.

Many previous studies investigated the derivation of the rain rate R ($\sim 3.67^{th}$ moment of the DSD) or the radar reflectivity factor Z (6^{th} moment of the DSD in the Rayleigh region, i.e. for the frequencies below 5 GHz approximately) from DSD; both quantities being not dependent on the frequency.

This contribution is focused on the discussion of the role of DSD on the rain attenuation at various frequencies in the region 10–150 GHz. A similar study could be performed in this frequency region for the radar reflectivity factor, which should be derived from the frequency dependent back scattering cross section and not from the 6^{th} DSD moment.

This study is based on the Mie scattering approximation for temperature of 5°C (this approximation holds only for spherical rain drop shape which is a quite good technical sim-

plification to derive the rain attenuation). The computation of attenuation is demonstrated using the average DSD from the 11 month measurement at the Institute of Atmospheric Physics in the Czech Republic (Fiser, Schoenhuber, Pesice, 2002).

2 Evaluation of the specific attenuation

The specific rain attenuation is given by the next general formula:

$$A = 4.3434 \cdot \lambda \cdot 10^3 \int_0^\infty \text{Im}f(D)N(D)dD[\text{dB/km}] \quad (1)$$

where

- A is the specific rain attenuation,
- f is the complex scattering function being strongly dependent on the frequency,
- D is the rain drop diameter,
- $N(D)$ is symbol for the drop size distribution (DSD),
- λ is the wave length.

Equation (1) shows that the imaginary part of scattering function is directly proportional to the specific rain attenuation. To study the influence of the scattering function we plotted the imaginary part of the scattering function $\text{Im}f$ at frequencies in the range from 10 to 150 GHz using the Mie scattering formulas (see for example Fiser, 1993). It was acknowledged that the attenuating role is increasing with the rain drop diameter at given frequency. The attenuation is also proportional to the frequency.

The slope of the $\text{Im}f(D)$ decreases with the increasing frequency for rain drop diameter above about 1 mm. It means practically that the relative role of small rain drops increases with the increasing frequency (see part 5). Trying to find the simple dependence on its diameter we derived numerically

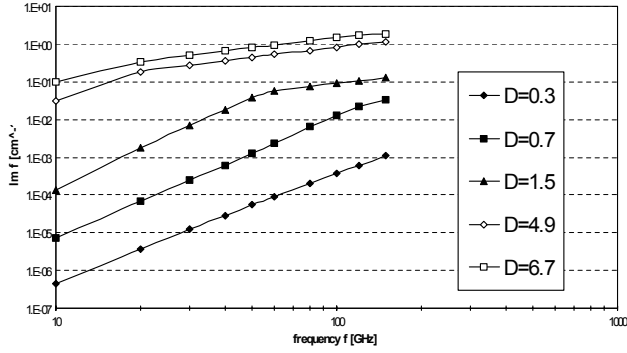


Fig. 1. The imaginary part of the forward scattering function $\text{Im } f$ (cm^{-1}) (Mie scattering). The rain drop diameter in (mm) is a parameter. This $\text{Im } f$ is proportional to the rain attenuation.

that the $\text{Im } f$ is approximately proportional to the 4^{th} power of the drop diameter at frequencies 10 to 30 GHz (it is close to the Rayleigh region) whereas in the frequency range 120–150 GHz the attenuating effect is approximately proportional to the 2^{nd} power of diameter (the drops achieve the optical region).

The imagination is obvious from Fig. 1. The slope of the attenuating role of larger drops decreases with the frequency.

On the other hand, the number of drops per unit volume and per rain drop interval (DSD) decreases with the increasing diameter. Both, DSD as well as the scattering function must be investigated jointly.

3 The drop size distribution

The rain attenuation is also proportional to the DSD $[N(D)]$ as it is obvious from equation (1). The general feeling about the DSD is that the $N(D)$ is rapidly decreasing with the drop diameter however it was found that the number of smaller drops is increasing with its diameter up to a certain threshold, in CZ it seems to be $D \sim 0.8$ mm. The analytical Gamma DSD model, for instance in the following form:

$$N(D) = N_0 D^\mu \exp(-\lambda^* D). \quad (2)$$

(N_0 , λ and μ are the intercept, slope and shape parameters, respectively) is able to approximate this fact, whereas simpler exponential DSD approximation can model only a decreasing function:

$$N(D) = N_0 \exp(-\lambda D). \quad (3)$$

4 Example of the DSD measurement

To show the influence of the rain drop diameter on the specific rain attenuation, the average DSD measured at the Institute of Atmospheric Physics in the Czech Republic was used (see Fiser, Schoenhuber, Pesice, 2002); its corresponding rain rate is 2.17 mm h^{-1}). The practical utilisation of all following conclusions may be limited to this measurement,

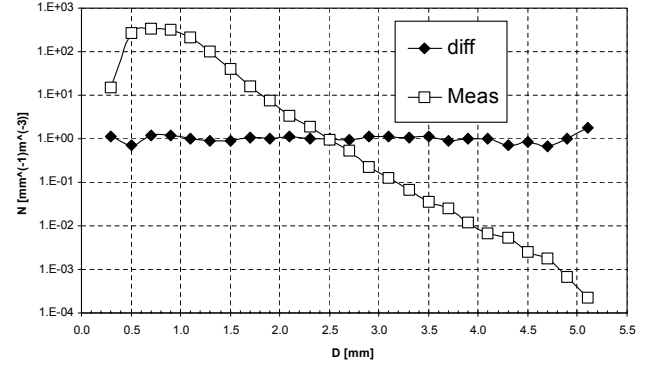


Fig. 2. DSD measured in the Czech Republic (27 July 1998 – 2 July 1999, curve “Meas”). The curve labelled “diff” shows the relative difference of the model “Step 2” described in the part 6.

however the used technique is general and the research is continuing.

This measurement was performed at the experimental site of the IAP in the town of Hradec Králové being 100 km east to Prague. The instrument was placed on the roof of the public astronomic observatory (50.18° N , 15.83° E , 285 m a.s.l.). The DSD measurement started on 27 July 1998 and finished on 2 July 1999. The number of nearly 2 millions (1 935 108) rain drops was recorded during the 11 months (340 day) experiment. Only the one-minute DSDs satisfying the condition that the rain rate exceeded 0.2 mm/h were considered. The average measured DSD (see Fig. 2) was also approximated by the Exponential (label E) and Gamma (label G) analytical function and the parameters were derived by two ways: by linear regression (label R) and by the method of moments (3^{rd} and 6^{th} moments of DSD were used in the Exponential model case while the 4^{th} moment was added to compute the Gamma model parameters; label M). For instance, E-M is a label for the Exponential model with parameters having been derived by the method of moments etc. A third method deriving the parameter μ consists in fixing the μ parameter to values 0, 1, 2...20 and deriving only the intercept N_0 and slope λ parameters through regression. This method is called “fix” method (label F). Only μ corresponding to the minimum RMSE is then considered.

The measurement results are shown in Table 1 (from Fiser, Schoenhuber, Pesice, 2002).

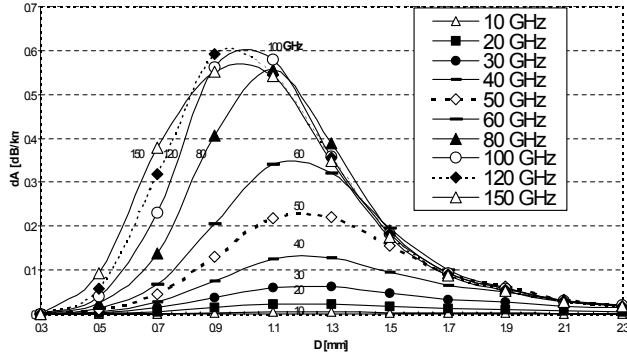
5 The role of particular rain drop size classes on specific rain attenuation

It is interesting to study the contribution of particular rain drops on the attenuation, i.e. the integrand of (1). Equation (1) shows that the particular contribution to the attenuation is given by the product of the imaginary part of the scattering function ($\text{Im } f$) and $N(D)$ (DSD).

The decreasing number of large rain drops predominates to the increasing attenuating role of the imaginary part of the scattering function and the integrand starts to decrease

Table 1. Parameters of the DSD models of the average DSD from the Czech Republic

Model	E-M	E-R	G-M	G-R	G-F
$N_o [mm^{-1} m^{-3}]$	4.0E+03	5.0E+02	5.0E+03	8.0E+02	2.0E+03
$\lambda [mm^{-1}]$	3.5	2.7	3.9	3	3.7
μ	0	0	0.5	0.7	2

**Fig. 3.** The “D” response of the integrand of Eq. (1) while the frequency is a parameter. Specific attenuation A is directly proportional to the surface area (quadratic) below the curve and above the x-axis.

for rain drop diameters above a certain threshold. When analysing higher frequencies this threshold will move towards smaller rain drops. Figure 3 is showing the integrand of (1) while the frequency is a parameter. This integrand includes all constants ($4.3434 \cdot \lambda \cdot 10^3$) from equation (1) thus the specific attenuation is directly proportional to the surface area (quadratic) below the curve and above the x-axis.

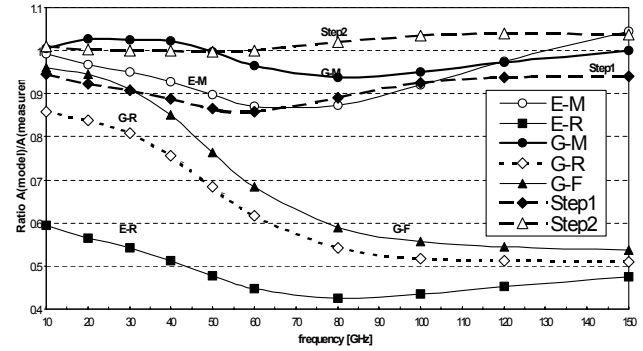
The maximum shifts towards smaller rain drops with the increasing frequency (maximum occurs at diameter $D \sim 1.2$ mm at 10 GHz whereas the maximum is found at the diameter $D \sim 0.95$ mm at 120 GHz). It is very interesting that the prevailing contribution to the specific attenuation is formed by drops of diameter not exceeding 2 mm! That’s way the analytical DSD models should be very accurate for drops below 2 mm of their diameter. Table 2 shows the percentage attenuation contribution formed by rain drops within the diameter interval $< 0.3; 1 >$ mm related to the total attenuation. The increasing role of small rain drops with the frequency is obvious.

6 Test of the analytical DSD models with respect to the rain attenuation computation

Using the measured average DSD (Fig. 2) we evaluated the rain rate (2.17 mm/h) and the attenuation at various frequencies in the range 10–150 GHz. The estimated attenuation was also computed when replacing the measured DSD by its approximations having been described in part 3 (the param-

Table 2. Percentage fraction of the attenuation formed by rain drops within the diameter interval $< 0.3; 1 >$ mm only

f (GHz)	10	20	30	40	50
fraction (%)	17.9	15.9	15.9	16.5	18.0
f (GHz)	60	80	100	120	150
fraction (%)	20.8	29.4	38.2	42.7	44.5

**Fig. 4.** Ratio of the rain attenuation derived through an analytical DSD model (G - Gamma, E - Exponential). M (R) represents the moment (regression) method used to derive the model parameters. Model “Step 1” and “Step 2” are models describing the DSD for 2 drop diameter intervals separately (see part 6).

eters N_o , λ [and μ in the Gamma DSD model case] were derived from the rain drop interval $< 0.3; 5.1 >$ mm). Figure 4 shows the ratio (deviation) of the specific rain attenuation derived from the measured DSD (Fig. 2) and DSD modelled by different DSD models. Both the method of moments as well as the linear regression were used.

One can see that the methods E-R, G-R and G-F are not suitable. The DSD models with parameters derived by the method of moments (Gamma model preferably) yields a much better fit. On the other hand, the DSD models with parameters derived by the moment method do not describe the natural DSD response (N-D dependence) accurately. This can be explained by the fact that moments of orders 3, 4 and 6 were used in our technique. The higher order moments (4^{th} and 6^{th}) are strongly dependent on larger raindrops and therefore the approximated $N(D)$ dependence is deviated in small rain drop area.

Having analysed the particular role of rain drops on the rain attenuation (see Fig. 3) we suggest to approximate the measured DSD by the 2-step model. The first step (only Gamma model) should derive the spectrum of rain drops within the diameter interval $< 0.3; 1.1 >$ mm. The spectrum for larger rain drops can be modelled exponentially. This model (see curve “Step 1” in Fig. 4) was not more accurate than the G-M model but its DSD description was much better than description by the G-M model. To improve this situation we enlarged the first diameter interval into $< 0.3; 1.3 >$ mm- this part was still in the “Gamma” region (found $N_o =$

$5.72 \cdot 10^7$, $\lambda = 12.17$, $\mu = 9.44$). The part for $D > 1.5$ mm was in the “Exponential region” although we modelled this second part also by the Gamma function which was slightly more accurate (found $N_o = 5.63 \cdot 10^3$, $\lambda = 4.13$, $\mu = 2.37$; see curve “Step 2” in Fig. 4 and then see Fig. 2) in comparison with the Exponential one.

7 Conclusion

This contribution shows the influence of the particular size of rain drops on the rain attenuation. It was shown, that

- the role of small drops is increasing with the increasing frequency
- the prevailing contribution to the specific attenuation is formed by drops of diameter not exceeding 2mm especially at higher frequencies considering our DSD example
- the moment methods are suitable to derive the parameters of DSD models for attenuation estimation but not for the modelling of the spectrum N-D response

- a 2 step model for the diameter intervals < 0.3 ; $1.3 >$ mm (Gamma) and < 1.5 ; $5.1 >$ mm (Gamma or Exponential) was suggested. This model is useful to compute the rain attenuation as well as it is suitable to model the spectrum (N-D response)

In the next work it will be worthy to study the changes being caused by the DSD at various rain rates, too.

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