

# A model of radar backscattering from the melting layer of precipitation

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**Abstract.** A theoretical model of backscattering of electromagnetic waves from the melting layer of precipitation is presented. It combines a simple microphysical model of the melting process with a T-matrix technique for computing the scattering of electromagnetic waves from collections of partially aligned, axially symmetric particles. The model can simulate backscattering by and propagation through the melting layer in the frequency range from 1 to 100GHz. Preliminary comparisons with Doppler radar observations at 3.3GHz are presented and discussed.

## 1 Introduction

The melting layer of precipitation is the atmospheric layer below the 0C isotherm where snowflakes melt and turn into raindrops. It is commonly known as the 'radar bright band' because it often appears as a band of enhanced reflectivity when viewed through radar operating at cm wavelengths. The microphysics of the melting layer of precipitation as well as its interaction with electromagnetic waves are important both in climate studies and in telecommunications: on the one hand, the melting layer is a characteristic feature of stratiform precipitation; on the other hand, it has a degrading effect on satellite communication links.

Studies of the melting layer of precipitation, either from a numerical, observational, or experimental point of view, abound in the literature; a summary of related work can be found in Szyrmer and Zawadzki (1999). The present study aims at developing a polarimetric model of scattering and absorption of electromagnetic waves from the melting layer of precipitation at mm wavelengths, a frequency range that is not covered by previously published Rayleigh-theory models (Russchenberg and Ligthart, 1996; D'Amico et al., 1998). To this end, a simple microphysical model of the melting process is combined with a T-matrix technique

for calculating the scattering of electromagnetic waves from partially aligned, axially symmetric particles (Skaropoulos, 2002; Skaropoulos and Russchenberg, 2002). Preliminary results from model calculations are shown to agree fairly well with Doppler radar observations at 3.3GHz.

## 2 Microphysical model of the melting layer of precipitation

The microphysical model is based on the simplifying assumptions that (a) the hydrometeors neither aggregate nor breakup during melting, and (b) the mass flux remains constant throughout the melting layer. These assumptions permit determining the size distribution of particles everywhere in the melting layer provided that the size distribution of raindrops below the melting layer and the density of snow particles  $\rho_s$  above the melting layer are known.

Just below the melting layer, the population of raindrops is assumed to follow a gamma size distribution (Ulbrich, 1983)

$$N_r(D_r) = N_0 D_r^\mu e^{-\Lambda D_r} [m^{-3} cm^1], \quad (1)$$

where  $D_r$  is the raindrop diameter in  $cm$ , and  $N_0 [m^{-3} cm^1]$ ,  $\mu$ , and  $\Lambda [cm^{-1}]$  are parameters of the distribution. Raindrops are assumed to fall with terminal velocity, as it is described by the empirical from of Atlas et al. (1973)

$$V_r(D_r) = \left( \frac{\rho_0}{\rho} \right)^{0.4} [9.65 - 10.3 \exp(-6D)], \quad (2)$$

with  $\rho_0$  and  $\rho$  being the air density at sea level and at the altitude of the melting layer bottom, respectively.

The size-distribution and fall-velocity characteristics of melting snowflakes are obtained from those of the resulting raindrops. First, through neglecting changes in the particle mass during melting, the diameter  $D_{ms}$  of a melting snowflake is related to the one of the raindrop by

$$D_{ms}^3 = D_r^3 [Q + (1 - Q)\rho_w/\rho_s], \quad (3)$$

where  $Q$  is the mass fraction of melted water and  $\rho_w$  and  $\rho_s$  are the mass densities of water and snow, respectively. Although the latter quantity depends, in general, on the snow crystal habit and size, it is treated as an independent of size, free parameter in the present model; its value determines to a large extent the excess reflectivity in the bright band, which is larger the smaller the snow density, and the width of the bright band, which is larger the larger the snow density. Second, the fall velocity of melting snowflakes  $V_{ms}$  is expressed as (Szyrmer and Zawadzki, 1999)

$$V_{ms}(D_{ms}, Q) = \frac{V_r(D_r)}{g(Q)} \quad (4)$$

where

$$g(Q) = 4.2 - 1.6Q - 1.6Q^2. \quad (5)$$

Finally, through invoking the assumption of constant mass flux, the number concentration  $N_{ms}$  of melting snowflakes can be obtained by the following relation:

$$N(D_{ms})V_{ms}(D_{ms}) = N(D_r)V_r(D_r). \quad (6)$$

The melting rate, that is, the dependence of the melted mass fraction  $Q$  on the depth  $h$  below the freezing level is modeled according to Mitra et al. (1990), who used wind tunnel experiments to derive the following thermodynamical differential equation:

$$\frac{dQ}{dh} = \frac{2\pi D_{ms}}{mL_m V_{ms}(D_{ms})} \bar{f}[k_a \Delta T + D_v L_e \Delta \rho_v]. \quad (7)$$

$m$  is the mass of the particle and quantities  $L_m$ ,  $\bar{f}$ ,  $k_a$ ,  $D_v$ ,  $L_e$  are defined in Mitra et al. (1990). The first term in the brackets corresponds to the heat deposited to the melting particle through conduction from the warmer surrounding air ( $\Delta T$  stands for the temperature difference), whereas the second term corresponds to the heat deposited to/removed from the particle by condensation/evaporation ( $\Delta \rho_v$  stands for the difference in water vapor density). Equation (7) can be solved numerically, as an initial value problem. It is worth noting that Eq. (7) holds, in a strict sense, only for spherical snowflakes. Nevertheless, numerical simulations not presented here have demonstrated that the effect of the particle shape on melting rate is small.

Finally, the shape and the orientation of the precipitating hydrometeors are modeled according to Russchenberg and Ligthart (1996): Melting snowflakes are assumed to be oblate spheroids with axial ratios that depend on their size and on their stage of melting. Furthermore, they are assumed to be partially aligned along the vertical direction, their orientation being described by an axially symmetric probability density function with variance depending on the stage of melting.

### 3 Electromagnetic scattering from the melting layer of precipitation

Melting particles are inhomogeneous mixtures of air, snow, and water, the relative fraction of each constituent depending on the stage of melting. For the purposes of this study,

**Table 1.** Settings of TARA atmospheric radar

Transmit power	0.34 Watt
Range resolution	15 m
Time resolution	1.536 s
Polarization state	HH
Elevation	90°

they are treated as homogeneous, and an effective dielectric constant is assumed, although in reality the situation may be much more complex (Mitra et al., 1990). The effective dielectric permittivity is obtained as the weighted average of the Maxwell-Garnett mixing formula for two different topologies (Russchenberg and Ligthart, 1996), the first assuming snow inclusions in a water matrix and the second assuming water inclusions in a snow matrix.

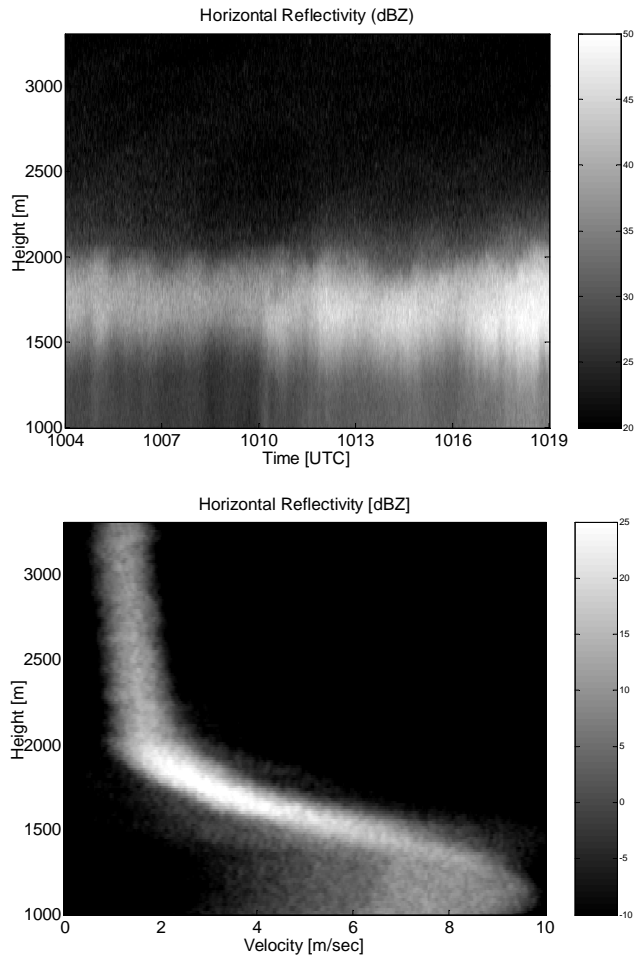
Because their concentration is small and their locations do not have any systematic relationship, particles in a volume element of the melting layer are assumed to scatter independently and incoherently. Therefore, scattering and absorption of electromagnetic waves from a collection of partially aligned melting hydrometeors can be described in terms of the ensemble-averaged Mueller matrix and the ensemble-averaged extinction matrix, respectively (Mishchenko et al., 2000). These quantities are calculated by use of the T-matrix approach and of a technique for averaging over particle orientations analytically; the relevant theory is described in detail in Skaropoulos (2002) and Skaropoulos and Russchenberg (2002). It is worth noting that the time-consuming computation of the T-matrix is done for only one orientation of the scattering particle, namely for the case where the symmetry axis of the particle is along the direction of the incident electromagnetic wave. The average over orientations is calculated analytically through use of the quantum theory of angular momentum. This technique can speed up computations by a factor of several tens (Mishchenko, 1991).

### 4 Comparison with radar observations at S band

Radar observations of the melting layer of precipitation were made on 19 September 2001, in the framework of the Baltex/Bridge Cloud Campaign (CLIWA-NET home page). During the measurement period (from 09:00 to 12:00 UTC) there was stratiform rain of moderate to hard intensity. The 0° isotherm was recorded by radio soundings to be at a height of approximately 2km.

The TARA atmospheric radar (Heijnen et al., 2000), developed and operated by the Delft University of Technology, was used. It is a 3.3GHz FM-CW Doppler-polarimetric radar, capable of recording the time series of the beat signal. The settings of the radar system during the measurement period are summarized in Table 1.

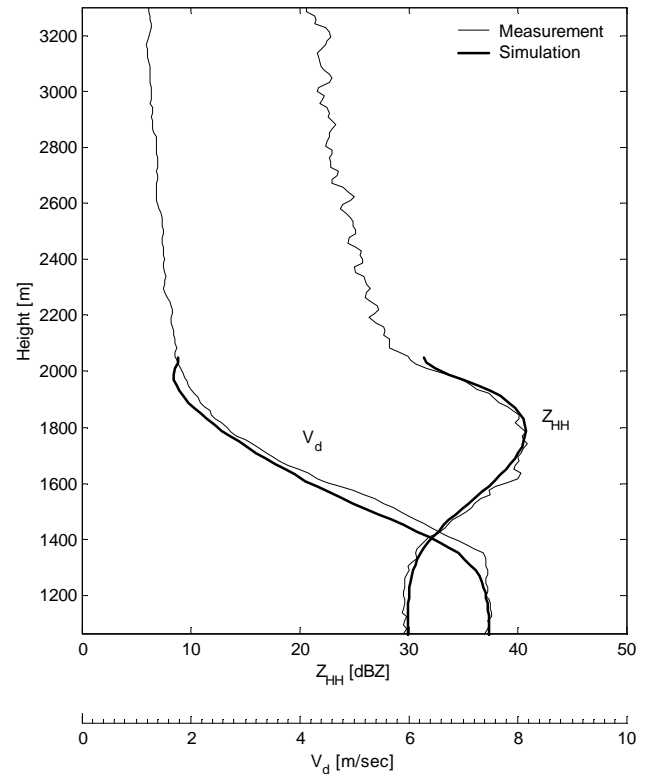
Results are presented in Fig. 1. The top panel depicts a 15-minute, time-height image of the equivalent horizontal re-



**Fig. 1.** Radar observations of the melting layer of precipitation (S band). (a) Time-height image of  $Z_{HH}$ , (b) Spectrograph of  $Z_{HH}$ .

flectivity factor. A band of enhanced reflectivity (from 2000 m up to approximately 1500 m) is clearly identifiable. Noticeable is also the increase of the peak reflectivity and of the width of the bright band during the measurement. The bottom panel depicts a 30-second averaged spectrograph of horizontal reflectivity, centered around 10:06 UTC. Each horizontal line corresponds to a Doppler spectrum at a particular height; the spectra have been unfolded and the convention is that positive velocities are approaching the radar. The spectra can be seen to widen and to change from symmetrical to negatively skewed as a result of the melting process.

Comparisons between simulations and observations are presented in Fig. 2. Plotted are the horizontal equivalent reflectivity factor,  $Z_{HH}$ , and the mean Doppler velocity  $V_d$ . The values of the various model parameters are given in Table 2. The DSD parameters in rain have been selected by fitting the simulated moments of the Doppler spectrum to the measured ones, whereas the snow density has been chosen so that the simulated excess reflectivity in the bright band matches the measured one. The simulated results can be seen to agree well with the measurements, although this is partly due to the fact that the model has many free parameters and,



**Fig. 2.** Simulated and measured profiles of radar observables.

thus, it can be, more or less, easily tuned to a single observation. A more comprehensive comparison with observations is necessary for checking whether the model can reproduce a variety of melting layer events.

## 5 Conclusions and future work

A theoretical model that combines microphysics of the melting layer of precipitation with a T-matrix method for the scattering of electromagnetic waves by partially aligned axially symmetric particles has been presented. The model is intended for simulating the scattering and the absorption of electromagnetic waves from the melting layer of precipitation in the frequency range from 1 to 100GHz. Preliminary comparisons with Doppler radar observations at S band have demonstrated that the model is capable of reproducing the observations.

Future plans include a more comprehensive assessment of the model via comparisons (a) at the level of Doppler spectra, (b) with dual-polarized observations, and (c) with observations at 35 and at 94GHz. Doppler spectrum comparisons can be used for refining the fall-velocity modeling, and dual-polarization comparisons can provide information on the size-shape dependence and on the orientation of the melting hydrometeors. In addition, combining Doppler with polarimetric information, as in Verlinde et al. (2002) and Hassiotis et al. (2002), can bring details of the melting process into light. Finally, comparisons at higher frequencies

**Table 2.** Model parameters for Fig. 2

Frequency	3.3 GHz
Elevation angle	90°
$N_0$	$2.92 \times 10^6 \text{ cm}^{-1} \text{ m}^{-3}$
Dispersion factor $\mu$	2.2
Slope of DSD $\Lambda$	$36 \text{ cm}^{-1}$
Snow density	$0.070 \text{ gr/cm}^3$
Adiabatic lapse rate	$6^\circ \text{ C km}^{-1}$
Orientation width (snow)	90°
Orientation width (rain)	3°
Minimum axial ratio	0.3
Maximum axial ratio	0.9

may assist in refining the modeling of the effective dielectric properties: such comparisons will also require incorporating into the model the attenuation of the electromagnetic waves in their propagation path. Eventually, the refined model will be used for predicting the attenuation and the depolarization of electromagnetic waves through the melting layer as well as the effect of the melting layer on ground-based and spaceborne, active and passive, algorithms for the retrieval of precipitation.

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