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# Estimating errors in the velocities obtained by a multiple bistatic Doppler radar

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**Abstract.** Error estimates of the horizontal velocity vector obtained using one Doppler radar and two receivers are given. Three Doppler velocities are combined to give two components of the horizontal velocity. This problem is composed of an overdetermined equation system, which has no unique solution. Therefore among an infinite number of solutions, those in which the error is smallest should be selected for practical use. This paper examines three methods: the simple, maximum likelihood, and least squares methods. The magnitudes of the errors were found to be in the following order: least squares method < maximum likelihood method < simple method.

## 1 Introduction

Multiple bistatic Doppler radar systems have two advantages over multiple-Doppler radar systems. First, a single Doppler radar illuminates whole precipitating system, so that the Doppler radar and receivers observe any area at the same instance. Second, the system is much cheaper than a multiple Doppler radar system. However, in a system that is composed of a radar and a receiver, the data near the baseline have large errors (Wurman et al., 1993; de Elia and Zawadzki, 2001; Takaya and Nakazato, 2002). To avoid this problem, it is recommended that at least two receivers should be used.

This paper examines a multiple bistatic Doppler radar system composed of a Doppler radar and two receivers. In this case, there are three Doppler velocity data. Any pair of Doppler velocities gives a velocity vector. Therefore, this system can generate three velocity vectors. This paper evaluates which combination of these three vectors gives the minimum root mean square error (RMSE).

For simplicity, we consider the case in which the elevation angle of the radar beam is 0 degrees, and examine problems that can be solved in a horizontal plane.

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## 2 Geometrical Configuration

Consider a triangle 012, with the radar located at point 0 (See Fig. 1a); the remaining two points are the locations of receivers 1 and 2, respectively. Point $P$ represents a point where a scatterer is located. Three vectors $\boldsymbol{e_0}$, $\boldsymbol{e_1}$, and $\boldsymbol{e_2}$ of unit length parallel the coordinate vectors $\boldsymbol{0P}$, $\boldsymbol{1P}$, and $\boldsymbol{2P}$, respectively. The unit vectors $\boldsymbol{e_{01}}$ and $\boldsymbol{e_{20}}$ bisect angles $0P1$ ($\phi_{01}$ hereafter) and $2P0$ ($\phi_{20}$), respectively. Angle $1P2$ is represented by $\phi_{12}$. The directions of unit vectors $\boldsymbol{e_0}$, $\boldsymbol{e_{01}}$, and $\boldsymbol{e_{20}}$ are parallel to the Doppler velocities of the radar, receiver 1, and receiver 2, respectively (Protat and Zawadzki, 1999).

## 3 Combining the three velocity vectors

The combination of the Doppler radar and receiver 1 gives velocity vector $\boldsymbol{V_{01}}$, while the combination of the Doppler radar and receiver 2 gives velocity vector $\boldsymbol{V_{20}}$ and that of receivers 1 and 2 gives velocity vector $\boldsymbol{V_{12}}$. In the overdetermined observation domain, retrieved velocity vector is expressed by a linear combination of three Doppler velocities. This combination has another expression in terms of three retrieved velocity vectors in the following way:

$$\boldsymbol{V} = c_{01}\boldsymbol{V_{01}} + c_{12}\boldsymbol{V_{12}} + c_{20}\boldsymbol{V_{20}}\,. \tag{1}$$

The mean square error of this combined vector is given by:

$$\sigma^2 \equiv \overline{(\delta\boldsymbol{V}\cdot\delta\boldsymbol{V})}$$
$$= \left\{ \begin{array}{ll} c_{01}^2\overline{(\delta\boldsymbol{V}_{01}\cdot\delta\boldsymbol{V}_{01})} & +2c_{01}c_{12}\overline{(\delta\boldsymbol{V}_{01}\cdot\delta\boldsymbol{V}_{12})} \\ +c_{12}^2\overline{(\delta\boldsymbol{V}_{12}\cdot\delta\boldsymbol{V}_{12})} & +2c_{12}c_{20}\overline{(\delta\boldsymbol{V}_{12}\cdot\delta\boldsymbol{V}_{20})} \\ +c_{20}^2\overline{(\delta\boldsymbol{V}_{20}\cdot\delta\boldsymbol{V}_{20})} & +2c_{20}c_{01}\overline{(\delta\boldsymbol{V}_{20}\cdot\delta\boldsymbol{V}_{01})} \end{array} \right\} \tag{2}$$

Throughout this paper, $(\boldsymbol{A}\cdot\boldsymbol{B})$ means the scalar product of vectors $\boldsymbol{A}$ and $\boldsymbol{B}$, and $\delta\boldsymbol{A}$ is the error part of vector $\boldsymbol{A}$. Three of the above six correlation functions are the mean square errors of the three Doppler velocity vectors:

$$\sigma_{01}^2 = \overline{(\delta\boldsymbol{V}_{01}\cdot\delta\boldsymbol{V}_{01})} \tag{3.1}$$

$$\sigma_{12}^2 = \overline{(\delta\boldsymbol{V}_{12}\cdot\delta\boldsymbol{V}_{12})} \tag{3.2}$$

$$\sigma_{20}^2 = \overline{(\delta\boldsymbol{V}_{20}\cdot\delta\boldsymbol{V}_{20})} \tag{3.3}$$

### 3.1 Simple method

The simple method (SM) is based on the following identity:

$$\begin{aligned}&\boldsymbol{e}_0 d_0 + \boldsymbol{e}_{01}\frac{d_1}{\cos\left(\frac{\phi_{01}}{2}\right)} + \boldsymbol{e}_{20}\frac{d_2}{\cos\left(\frac{\phi_{20}}{2}\right)}\\ &\quad = \boldsymbol{e}_0(\boldsymbol{e}_0\cdot\boldsymbol{V}) + \boldsymbol{e}_{01}(\boldsymbol{e}_{01}\cdot\boldsymbol{V}) + \boldsymbol{e}_{20}(\boldsymbol{e}_{20}\cdot\boldsymbol{V}),\end{aligned} \tag{4}$$

where $d_0$, $d_1$, and $d_2$ are the Doppler velocities of the radar, receiver 1, and receiver 2, respectively. Solving this equation with respect to $\boldsymbol{V}$ gives the linear combination.

$$\begin{aligned}\boldsymbol{V}_{SM} &= \frac{1}{\sin^2\left(\frac{\phi_{01}}{2}\right) + \sin^2\left(\frac{\phi_{12}}{2}\right) + \sin^2\left(\frac{\phi_{20}}{2}\right)}\\ &\times\left[\sin^2\left(\frac{\phi_{01}}{2}\right)\boldsymbol{V}_{01} + \sin^2\left(\frac{\phi_{12}}{2}\right)\boldsymbol{V}_{12} + \sin^2\left(\frac{\phi_{20}}{2}\right)\boldsymbol{V}_{20}\right]\end{aligned} \tag{5}$$

Putting the error part of (5) into (2), we obtain the mean square error by the simple method:

$$\begin{aligned}\sigma_{SM}^2 &= \left[\frac{1}{\sin^2\left(\frac{\phi_{01}}{2}\right) + \sin^2\left(\frac{\phi_{12}}{2}\right) + \sin^2\left(\frac{\phi_{20}}{2}\right)}\right]^2\\ &\times\left\{\begin{array}{l}\sigma_0^2\left[\sin^2\left(\frac{\phi_{20}}{2}\right) + 2\cos\left(\frac{\phi_{12}}{2}\right)\sin\left(\frac{\phi_{20}}{2}\right)\sin\left(\frac{\phi_{01}}{2}\right) + \sin^2\left(\frac{\phi_{01}}{2}\right)\right]\\ +\frac{\sigma_1^2}{\cos^2\left(\frac{\phi_{01}}{2}\right)}\left[\sin^2\left(\frac{\phi_{01}}{2}\right) + 2\cos\left(\frac{\phi_{20}}{2}\right)\sin\left(\frac{\phi_{01}}{2}\right)\sin\left(\frac{\phi_{12}}{2}\right) + \sin^2\left(\frac{\phi_{12}}{2}\right)\right]\\ +\frac{\sigma_2^2}{\cos^2\left(\frac{\phi_{20}}{2}\right)}\left[\sin^2\left(\frac{\phi_{12}}{2}\right) + 2\cos\left(\frac{\phi_{01}}{2}\right)\sin\left(\frac{\phi_{12}}{2}\right)\sin\left(\frac{\phi_{20}}{2}\right) + \sin^2\left(\frac{\phi_{20}}{2}\right)\right]\end{array}\right\}\end{aligned} \tag{6}$$

In (6), $\sigma_k^2(k=0,1,2)$ are the variances of the Doppler velocities of the Doppler radar, receiver 1, and receiver 2, respectively. They can be assumed to be equal if the signal processors are the same.

It is easily seen that error becomes infinite on baselines 01 and 20 due to factors $1/\cos^2(\phi_{01}/2)$ and $1/\cos^2(\phi_{20}/2)$. The horizontal distribution of the RMSE is given in Fig. 1b by assuming the RMSEs of the three Doppler velocities to be 1 m/s.

### 3.2 Maximum likelihood method

The maximum likelihood method (MLM) requires that the weight is inversely proportional to the mean square error of each retrieved velocity vector. Therefore, the synthesized velocity vector becomes

$$\begin{aligned}\boldsymbol{V}_{MLM} &= \frac{1}{\frac{1}{\sigma_{01}^2} + \frac{1}{\sigma_{12}^2} + \frac{1}{\sigma_{20}^2}}\\ &\left(\frac{1}{\sigma_{01}^2}\boldsymbol{V}_{01} + \frac{1}{\sigma_{12}^2}\boldsymbol{V}_{12} + \frac{1}{\sigma_{20}^2}\boldsymbol{V}_{20}\right)\end{aligned} \tag{7}$$

Equation (7) and (2) are combined to give the mean square error in this method:

$$\begin{aligned}\sigma_{MLM}^2 &= \left(\frac{1}{\frac{1}{\sigma_{01}^2} + \frac{1}{\sigma_{12}^2} + \frac{1}{\sigma_{20}^2}}\right)^2\\ &\times\left[\begin{array}{l}\frac{1}{\sigma_{01}^2} + \frac{1}{\sigma_{12}^2} + \frac{1}{\sigma_{20}^2} + \frac{2}{\sigma_{01}^2\sigma_{12}^2}\overline{(\delta\boldsymbol{V}_{01}\cdot\delta\boldsymbol{V}_{12})}\\ +\frac{2}{\sigma_{12}^2\sigma_{20}^2}\overline{(\delta\boldsymbol{V}_{12}\cdot\delta\boldsymbol{V}_{20})} + \frac{2}{\sigma_{20}^2\sigma_{01}^2}\overline{(\delta\boldsymbol{V}_{20}\cdot\delta\boldsymbol{V}_{01})}\end{array}\right]\end{aligned} \tag{8}$$

The horizontal distribution of the RMSE is shown in Fig. 1c. The RMSEs of the three Doppler velocities are also assumed to be 1 m/s. Unlike the SM, the values on the baselines 01 and 20 are finite.

### 3.3 Least squares method

The least squares method (LSM) starts with the following equations:

$$U = C_{x0}d_0 + C_{x1}\frac{d_1}{\cos\left(\frac{\phi_{01}}{2}\right)} + C_{x2}\frac{d_2}{\cos\left(\frac{\phi_{20}}{2}\right)} \tag{9.1}$$

$$V = C_{y0}d_0 + C_{y1}\frac{d_1}{\cos\left(\frac{\phi_{01}}{2}\right)} + C_{y2}\frac{d_2}{\cos\left(\frac{\phi_{20}}{2}\right)} \tag{9.2}$$

In (9), $U$ and $V$ are the $x$- and $y$-components of horizontal velocity vector $\boldsymbol{V}$. $d_0$, $d_1$, and $d_2$ are the Doppler velocities of the radar, receiver 1, and receiver 2, respectively. $C_{x0}$, $C_{y0}$, $C_{z0}$, $C_{x1}$, $C_{y1}$ and $C_{z1}$ are unknown coefficients that are to be determined by the Lagrange's method of indeterminate coefficients so that the mean square error of $\boldsymbol{V}$ becomes a minimum with satisfying certain orthogonal conditions. The resultant equation is given by

$$\boldsymbol{V}_{LSM} = \frac{\sigma_0^2\sin^2\left(\frac{\phi_{12}}{2}\right)\boldsymbol{V}_{12} + \frac{\sigma_1^2}{\cos^2\left(\frac{\phi_{12}}{2}\right)}\sin^2\left(\frac{\phi_{20}}{2}\right)\boldsymbol{V}_{20} + \frac{\sigma_2^2}{\cos^2\left(\frac{\phi_{20}}{2}\right)}\sin^2\left(\frac{\phi_{01}}{2}\right)\boldsymbol{V}_{01}}{\sigma_0^2\sin^2\left(\frac{\phi_{12}}{2}\right) + \frac{\sigma_1^2}{\cos^2\left(\frac{\phi_{01}}{2}\right)}\sin^2\left(\frac{\phi_{20}}{2}\right) + \frac{\sigma_2^2}{\cos^2\left(\frac{\phi_{20}}{2}\right)}\sin^2\left(\frac{\phi_{01}}{2}\right)}. \tag{10}$$

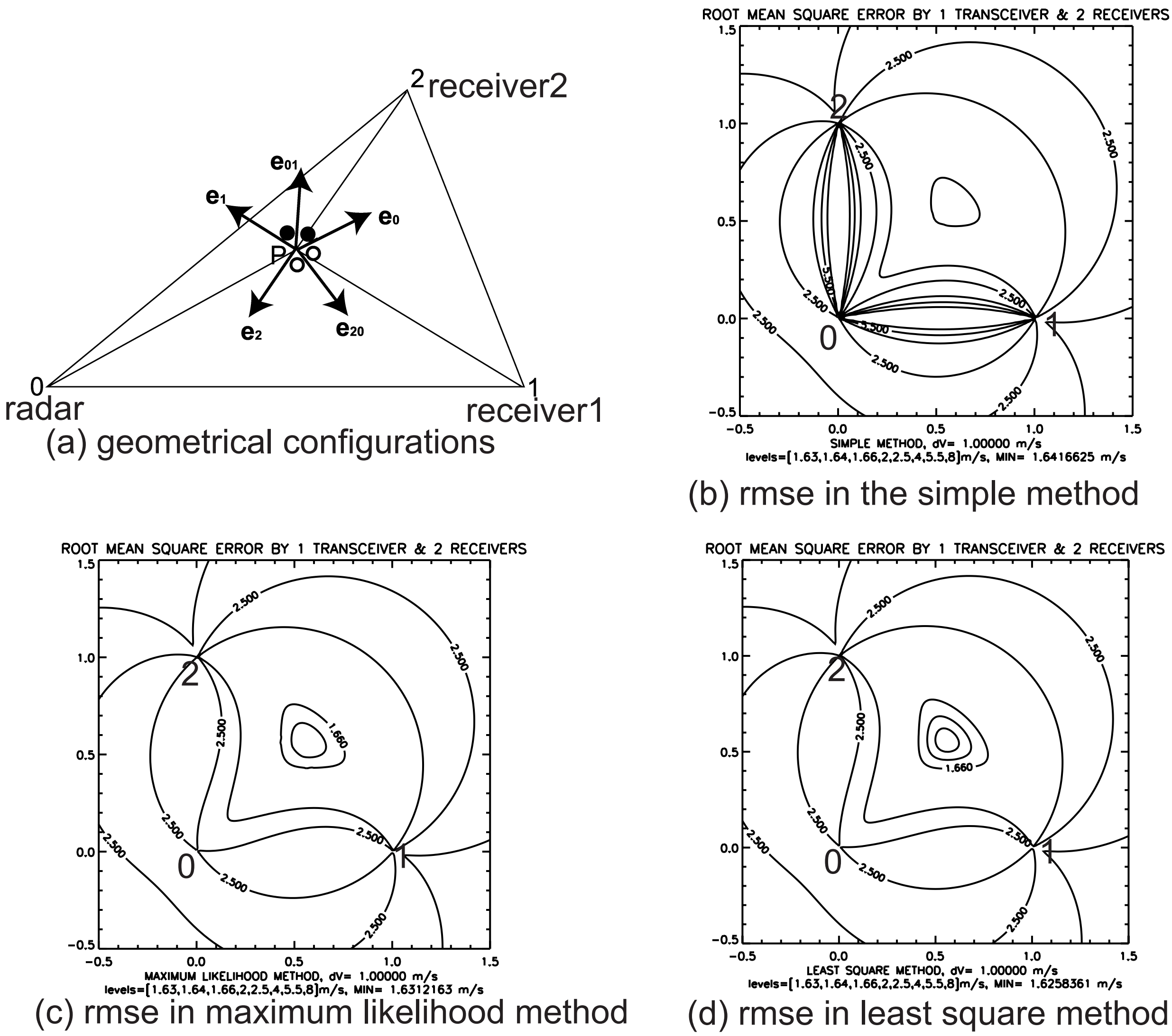


**Fig. 1. (a)** Geometric configuration of a multiple bistatic Doppler radar system composed of one Doppler radar and two receivers. The radar is located at point 0, and the two receivers are at points 1 and 2. A scatterer is located at point P. Unit vectors e1, e01, and e20 are parallel to the Doppler velocities of the radar, receiver 1, and receiver 2, respectively. **(b)**, **(c)**, and **(d)** show the horizontal distribution of the RMSE of the SM, MLM, and LSM, respectively. The RMSEs of the Doppler velocities are all assumed to be 1 m/s. The horizontal scale is normalized with respect to the baseline. In Fig. 1b, the RMSE goes to infinity on the baseline. In the other two methods, the RMSE always remains finite. The LSM realizes the smallest value of 1.626 m/s.

The mean square error in this case becomes

$$\sigma_{LSM}^2 = \frac{\sigma_0^2\left[\frac{\sigma_1^2}{\cos^2\left(\frac{\phi_{01}}{2}\right)}+\frac{\sigma_2^2}{\cos^2\left(\frac{\phi_{20}}{2}\right)}\right]+\frac{\sigma_1^2}{\cos^2\left(\frac{\phi_{01}}{2}\right)}\frac{\sigma_2^2}{\cos^2\left(\frac{\phi_{20}}{2}\right)}}{\sigma_0^2\sin^2\left(\frac{\phi_{12}}{2}\right)+\frac{\sigma_1^2}{\cos^2\left(\frac{\phi_{01}}{2}\right)}\sin^2\left(\frac{\phi_{20}}{2}\right)+\frac{\sigma_2^2}{\cos^2\left(\frac{\phi_{20}}{2}\right)}\sin^2\left(\frac{\phi_{01}}{2}\right)}. \tag{11}$$

The horizontal distribution of the RMSE is depicted in Fig. 1d. In this case, the minimum value is the smallest among the three methods.

## 4 Conclusions

Three methods to obtain a Doppler velocity vector by synthesizing three Doppler velocities were compared. Of these, the least squares method realized the smallest minimum value for the root mean square error. Moreover, by making point-by-point comparison within the domain of our analysis, the magnitudes of the errors were found to be in the following order: least squares method > maximum likelihood method > simple method. Especially, in the simple method, root mean square error became infinite on the baselines. Therefore, we recommend using the least squares method.

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