Proceedings of ERAD (2002): 405–408 © Copernicus GmbH 2002

ERAD 2002

Observation operator for Doppler radar radial winds in HIRLAM 3D-Var

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Abstract. Observation operator for the Doppler radar radial winds has been developed for the HIRLAM 3D-Var. The first version of the observation operator involved a linear interpolation of the model u- and v- wind components to the observation location and a projection of the interpolated wind components towards the radar on the slanted direction of the radar beam. In reality the radar beam broadens with increasing range from the radar. It is thus tempting to use a gaussian weighting of the horizontal model wind profile at the observation location rather than a linear interpolation from the two nearest model levels. The effect of the radar beam bending is studied by calculating the local refraction index gradient from the model temperature, humidity and pressure profiles.

1 Introduction

The scope of this paper is the assimilation of Doppler radar radial winds into numerical weather prediction model. Wind observations are particularly important in forecasting quickly developing mesoscale systems, because in these scales geostrophic adjustment is weak and the mass field adjusts to the wind field (Rinne 2000). Doppler radar wind observations constitute a potential source of wind information.

The first version of the observation operator for radar radial winds and a 10-day assimilation experiment with HIRLAM (HIgh Resolution Limited Area Model) 3D-Var were validated and reported in the previous ERAD meeting in Bologna Italy, 2000 (Lindskog et al. 2000). In this paper, the fit of the model counterpart with observations is studied in more detail and the observation operator is further developed to be more faithful to the nature of radar wind measurements. Section 2 describes the basics of the observation operator. Development and testing is considered in Sect. 3 and a brief summary is given in Sect. 4.

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2 Observation operator

Three dimensional variational data assimilation (3D-Var) is based on the minimization of the cost function

$$J = J_b + J_o = \frac{1}{2}(\mathbf{x}-\mathbf{x^b})^T\mathbf{B}^{-1}(\mathbf{x}-\mathbf{x^b})$$
$$+\frac{1}{2}(H\mathbf{x}-\mathbf{y})^T\mathbf{R}^{-1}(H\mathbf{x}-\mathbf{y}), \quad (1)$$

where J_b measures the distance of the model state vector $\mathbf{x}$ to the background model state vector $\mathbf{x^b}$ and J_o to the observation vector $\mathbf{y}$ respectively (Gustafsson et al. 2001). Observation operator H produces the model counterpart of the observed quantity.

The observation operator for radial winds (Lindskog et al. 2000) involves first linear interpolation of model data to the observation location. Then a projection of the horizontal model wind towards the radar at the observation point is calculated with

$$v_h = u\sin\theta + v\cos\theta, \quad (2)$$

where u and v are the model horizontal wind components in x- and y- direction and θ is the azimuth angle of the radar beam. The v_h is finally projected on the slanted direction of the radar beam

$$v_r = v_h\cos(\phi+\alpha)$$

where

$$\alpha = \arctan\left(\frac{d\cos\phi}{d\sin\phi+\frac{4}{3}r+h}\right) \quad (3)$$

and ϕ is the elevation angle of the radar beam. The formula for α takes approximately into account the curvature of the Earth. In term α, d is the measurement range, r is the radius of the earth and h is the height of the radar above the mean sea level.

There are three major assumptions in this formulation of the observation operator. First, the radar beam broadening is not taken into account. Second, the bending of the radar beam is not properly taken into account. Third, it is assumed that there is no mean velocity towards the radar due to precipitation. The last assumption is valid only for measurements with low elevation angles.

3 Further observation operator development and testing

3.1 Gaussian weight function

The radar beam broadens with increasing range and the wind observation represents the larger measurement volume the longer is the measurement range. The shape of the radar beam main lobe can be approximated with a gaussian function (Probert-Jones 1962). The elementary solution to model the broadening of the radar beam in the observation operator is thus to use a gaussian weight function

$$w = \frac{1}{2\pi} \exp\left(-\frac{(z-z_0)^2}{\kappa}\right) \quad (4)$$

in vertical instead of linear interpolation when defining the model u- and v- components to the observation height. In (4) z is the model level height and z_0 the observation height. The κ-term defines the width of the filter response function.

The gaussian weight function is nonzero from the earth's surface to the top of the atmosphere. The wind information which the radar is unable to measure should not be used when defining the model counterpart. Radars are unable to see below certain height, i.e. below the radar horizon. The obscuring effect of the radar horizon is taken into account by assuming radar horizon of 0° below which the model information is not used.

The observation operator assumes a homogeneous field of scatterers between the radar horizon and the top of the model domain. The empirical upper integration limit is set to 1.5 times the beamwidth (Jarmo Koistinen, personal communication). This is based on the fact that the radar reflectivity usually decreases rapidly above that height. Also, above this height the gaussian weight is already so small that the information does not have a notable effect to the calculated wind component.

Figures 1a, 1b and 1c show examples of the gaussian weight function at different measurement ranges. When the measurement range is short the vertical extent of the weight function is narrow, and it broadens with the increasing range.

If the vertical wind profile is linear, the model counterpart is the same when calculated by the linear interpolation or by the gaussian weight function. The effect of using the gaussian weight function as opposed to the linear interpolation is the largest when the wind profile is nonlinear and there is a local maximum or minimum near the observation height. This is illustrated by Fig. 1d. In linear interpolation the model counterpart would be calculated from the two nearest model levels, and the interpolated wind component would also be near the actual local maximum/minimum of

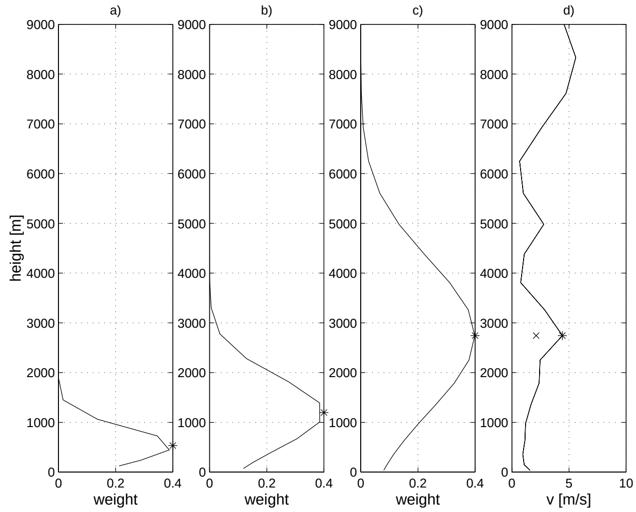

Fig. 1. The shape of the gaussian weight function (solid line) in vertical with measurement ranges **(a)** 15 km ($\kappa = 2.7 \times 10^5$ m), **b)** 45 km ($\kappa = 1.1 \times 10^6$ m) and **(c)** 85 km ($\kappa = 5.2 \times 10^6$ m), observation height marked with *. **(d)** Model wind profile for v-component (solid line). Wind at observation height marked with * when calculated with linear interpolation and with x when calculated applying gaussian weight function in vertical.

the model wind profile. The gaussian weight function integrates over several model levels and therefore the model counterpart deviates from the actual model wind profile.

A 14 day (1–14 June 1999) experiment has been performed to study the difference between the radial superobservations and the model background, which is a 6 hour forecast. Superobservations are spatial averages of raw measurements. They represent better the horizontal model resolution than the raw observations. In order to evaluate the impact of the gaussian weight function the experiment has been performed both for the standard observation operator (linear interpolation) and for the improved observation operator (gaussian weight function). The observations are from the SMHI (Swedish Meteorological and Hydrological Institute) radar network, which has a large unambiguous velocity interval (±48 m/s).

Figure 2 displays the mean and RMS difference between observations and background as a function of height for the standard observation operator (solid line). Above 750 m height the mean difference increases implying that the model counterpart is always stronger than the observed wind. The height dependence of the mean difference means also range dependence, because the higher is the measurement height the longer is the measurement range. Dashed line of Figure 2 displays the mean and RMS difference between observations and background for the improved observation operator. Mean difference is slightly smaller for almost all heights in this case. The RMS difference profiles are almost identical for both runs. The overall impact of the gaussian weight function is positive.

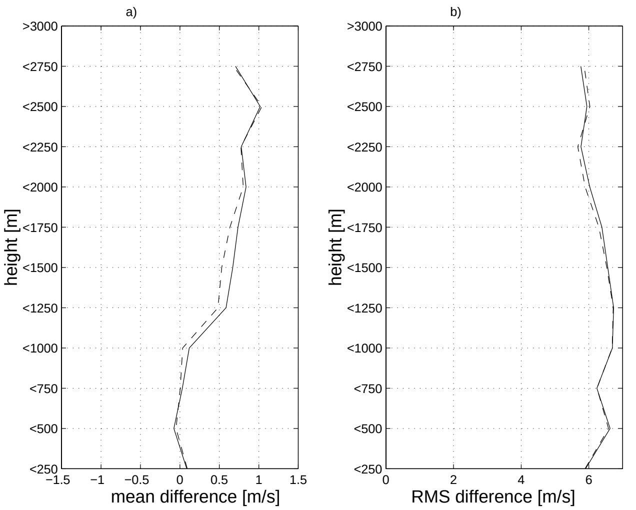

Fig. 2. Mean **(a)** and RMS **(b)** difference between observations and background as a function of height with solid line when linear interpolation is used in the observation operator and with dashed line when the gaussian weight function is used in the observation operator.

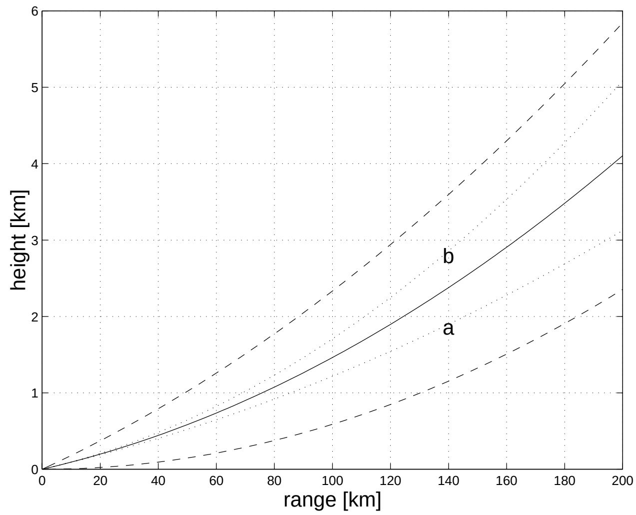

Fig. 3. Height of the radar beam shown as a function of measurement range in normal conditions (i.e. $\frac{4}{3}r$-law) (solid line), when the gradient of the refraction index is $\frac{dn}{dh} = -90 \times 10^{-6}$ (dotted line a) and when the gradient of the refraction index is $\frac{dn}{dh} = 10 \times 10^{-6}$ (dotted line b). Broadening of the radar beam is shown with dashed lines, when the beamwidth is 1° and the elevation angle 0.5°.

3.2 Bending of the radar beam

Using the gaussian weight function decreases the mean difference between observations and background, but the improvement is quite small. Another source of positive mean difference could be that the radar beam path is assumed to be too high, because the bending of the radar beam in the atmosphere is not properly taken into account.

The term α in the observation operator takes approximately into account the curvature of the earth. Doviak and Zrnic (1993) show that the radar ray path can be considered a straight line when an equivalent earth's radius

$$r_e = \frac{r}{1 + r\frac{dn}{dh}} \tag{5}$$

is used. In (5) r is the earth's radius and $\frac{dn}{dh}$ is the gradient of the refraction index. Typically the gradient of the refraction index is -40$\times10^{-6}$ km^{-1}. This value leads to an effective radius of approximately $r_e = \frac{4}{3}r$ which is used in (3). When the temperature increases with altitude, i.e. an inversion is present, downward bending of the radar rays is stronger than in normal conditions (ICAO atmosphere) and the gradient of the refraction index is smaller. This *super refraction* is typical in the lower parts of the atmosphere. If the gradient of the refraction index is larger than -40$\times10^{-6}$ km^{-1}, the radar beam is not bent downward as strongly as usual. In subrefraction conditions the effective earth radius is larger than $\frac{4}{3}r$.

Figure 3 illustrates radar beam height with increasing range in normal conditions $\frac{dn}{dh} = -40\times10^{-6}$ km^{-1}, when super refraction is present $\frac{dn}{dh} = -90\times10^{-6}$ km^{-1} (case a) and when subrefraction is present $\frac{dn}{dh}$ =10$\times10^{-6}$ km^{-1} (case b). Broadening of the radar beam is also illustrated assuming a beamwidth of 1°. For example at a range of 100 km the width of the radar beam main lobe is approximately 2 km. If the gradient of the refraction index is decreased from normal conditions to -90$\times10^{-6}$ km^{-1} the height of the radar beam differs from the $\frac{4}{3}r$-law by about 300 m.

In order to evaluate the impact of accounting the bending of the radar beam, the 14 day experiment has been repeated with the further modified observation operator. The local refraction index

$$n = \left(\frac{77.6}{T}(p + 4810\frac{e}{T})\right) \times 10^{-6} + 1, \qquad e = \frac{qp}{0.622} \tag{6}$$

is calculated from the model (T, q, p)-profile for model levels assuming horizontal homogeneity between the observation location and the radar location. In (6) T is temperature, p is pressure, e water vapour pressure and q specific humidity. The gradient of the refraction index is calculated as a pressure weighted average from the 10 lowest model levels. The effective radius of the earth is calculated by (5) and substituted into (3). The effect of calculating the local gradient of the refraction index is small on mean and RMS difference between observations and background, at largest of the order $\pm10^{-4}$ (no figure). This can be understood by studying the equation (3). If the gradient of the refraction index $\frac{dn}{dh}$ changes from -40$\times10^{-6}$ km^{-1} for example to -90$\times10^{-6}$ km^{-1} and the measurement range varies between 0 and 150 km, the change in $\cos(\phi + \alpha)$-term varies between 0 and 10^{-4}. This is exactly the order of magnitude of the change in calculated v_r.

4 Summary and conclusions

Observation operator for the Doppler radar radial winds is tested and further developed for HIRLAM 3D-Var. The difference between the observations and the model counterpart is studied. There is a positive height (range) dependent mean difference between observations and background of up to 1.0 m/s in the data, model counterpart being stronger than the observed wind. This mean difference can be reduced by up to 0.2 m/s, if a gaussian weight function is used when calculating the model u- and v- components to the observation height. Calculating the local gradient of the refraction index from model (T, q, p)-profile has no notable effect on mean and RMS difference. It seems to be accurate enough to use the $\frac{4}{3}r$-law when accounting for the bending of the radar beam.

Acknowledgement. The author wishes to express her acknowledgements to Heikki Järvinen and Jarmo Koistinen for supervision.

References

Doviak, R. J. and D. S. Zrnic: Doppler radar and weather observations. Second edition, San Diego Academic Press, Inc., 562 pp., 1993.

Gustafsson, N., Berre, L., Hörnquist, S., Huang, X.-Y., Lindskog, M., Navascués, B., Mogensen, K.S., and Thorsteinsson, S.: Three-dimensional variational data assimilation for a limited area model. Part I: General formulation and the background error constraint, Tellus, 53A, No. 4, 425–446, 2001.

Lindskog, M., Järvinen, H. and Michelson, D. B.: Assimilation of Radar Radial Winds in the HIRLAM 3D-Var. Phys. Chem. Earth (B), 25, No. 10–12, 1243–1249, 2000.

Probert-Jones, J. R.: The radar equation in meteorology. Quart. J. Roy. Meteor. Soc., 88, 485–495, 1962.

Rinne, J.: The use of radar wind observations in numerical assimilation. Phys. Chem. Earth, 25, 10-12, 1273–1276, 2000.