Proceedings of ERAD (2004): 233–236 © Copernicus GmbH 2004

ERAD 2004

Z-R (Radar Reflectivity-Rain rate) relationships derived from Czech Distrometer data

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Abstract. This study is trying to discuss the accuracy of selected Z-R relationships through the RMSE and correlation and to improve the accuracy using distinguished rain (DSD) types.

The Czech DSD data (one year) were used to derive the radar reflectivity factor Z as well as the rain rate. A better accuracy (lower RMSE) was observed when using different constants in the Z-R relationship for different DSD types. We used mathematicall DSD type definitions and the Waldvogel definition seems to be very promissing.

1 Introduction

It is practical to approximate the Z-R relationships by an analytical, simple and accurate function in order to estimate the rain rate R from the radar reflectivity factor Z measurement or vice versa. This contribution aims at finding parameters (two or three) of different Z-R approximations (Sect. 3). All shown Z-R relationships are based on the Czech distrometer data and also with respect to the rain type or drop size distribution (DSD) type.

Marshall and Palmer published the Z-R approximation firstly. In the past there was an effort to improve the accuracy of Z-R relations, for instance in Sokol et al. (2002).

This study is trying to discuss the accuracy of found Z-R relationships through the RMSE and correlation and to improve the accuracy using distinguished rain (DSD) type.

The radar reflectivity factor Z as well as the rain intensity R were derived from the actual DSD mathematically. These computed values can be considered in the sense of same values, which might be measured or observed by meteorological radar or by the rain gauge. So we can compute the radar reflectivity factor z (or Z in dBZ) from the DSD by following equation:

$$z = \int D^6 N(D) \mathrm{dD} [\mathrm{mm}^6 \mathrm{m}^{-3}] \tag{1}$$

$$Z = 10 logz [\mathrm{dBZ}] \tag{2}$$

where $N(D)$ is the drop size distribution (rain spectrum) and D is rain drop diameter.

Similarly, the rain rate R depends on the DSD through the well known equation:

$$R = \frac{3.6}{10^3} \pi \int_0^\infty D^3 v(D)\, N(D) dD \tag{3}$$

where $v(D)$ is the falling velocity of the drop of diameter D. The falling velocity is approximated by the Best's formula:

$$v(D) = 9.65 - 10.3 \exp(-0.6D) \tag{4}$$

2 Description of the source data

All results in this contribution were derived from the actual drop size distributions measurement with the videodistrometer of ESA, which was lent to the Institute of Atmospheric Physics Prague in the period July 1998–July 1999. The drop size distributions were related to the one-minute intervals (the time of integration was one minute). Only data for the rain intensity R being greater or equal to 0.2 mm/h were used.

The measurement in the Czech Republic was running at the experimental site of the IAP in the town of Hradec Králové (100 km east from Prague, see Fišer et al., 2002). The instrument was placed on the roof of the public astronomic observatory (50.18° N, 15.83° E, 285 m a.s.l).

After filtering the one minute DSDs were evaluated by calculating the number of drops in drop diameter classes 0–0.2, 0.2–0.4,…, 9.6–9.8 mm.

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Table 1. (all 4182 spectras) Found parameters of Z-R approximations (5–9) using all the Czech DSD data.

Type	a (or x)	b (or y)	q	RMSE[%]	corr.
(5)	229	1.278	0.0000	12.08	0.78
(6)	225	1.282	0.0596	12.10	0.79
(7)	226	1.258	0.0089	12.09	0.79
(8)	$> 10^{10}$	5.016	−0.2438	158.15	0.00
(9)	0	262.788	5.7009	2275.78	0.79

Table 2. CS >1 (1061 spectras) Found parameters of Z-R approximations (5–9) using selected Czech DSD data

Type	a (or x)	b (or y)	q	RMSE[%]	corr.
(5)	134	1.164	0.0000	8.91	0.89
(6)	128	1.169	0.1153	8.95	0.92
(7)	129	1.112	0.0207	8.93	0.95
(8)	$> 10^{10}$	4.428	−0.2087	133.37	0.07
(9)	$> 10^{10}$	61.776	25.2877	35432.11	0.00

3 Selected approximations of the Z-R relationship

We studied the accuracy of next selected Z-R relationships: a, power-law Z-R relationship was used by Marshal-Palmer. Its form is

$$z = a\,R^{\,b} \tag{5}$$

b, improved power-law Z-R relationship

$$z = x\,R^{\,(y + q\ \log R)} \tag{6}$$

c, modificated power-law Z-R relationship

$$z = x\,R^{\,(y + q\,R)} \tag{7}$$

d, second order polynomial relationship

$$z = x + yR + q\,R^2 \quad [\mathrm{mm^6 m^{-3}}] \tag{8}$$

e, second order polynomial relationship

$$Z = x + yR + q\,R^2 \quad [\mathrm{dBZ}] \tag{9}$$

To check the accuracy of the found form of the Z-R relationship both the correlation coefficient as well as the root mean square error (RMSE) were computed. For the RMSE we use:

$$RMSE = 100\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{Z_{appr} - Z_{meas}}{Z_{meas}}\right)^2} \tag{10}$$

where Z_{meas} is derived from the DSD using Eqs. (1) or (2).

Table 3. CS < 1 (3121 spectras) Found parameters of Z-R approximations (5–9) using selected Czech DSD data

Type	a (or x)	b (or y)	q	RMSE[%]	corr.
(5)	275	1.316	0.0000	10.85	0.83
(6)	271	1.320	0.0510	10.88	0.83
(7)	272	1.298	0.0087	10.87	0.83
(8)	$> 10^{10}$	5.191	−0.2504	148.69	0.00
(9)	0	325.409	43.3685	2366.05	0.83

4 Result 1: comparison of Z-R approximations using all measured DSDs

As one can see from Table 1, using the Czech data the derived power law Z-R relation is

$$Z = 229\,R^{\,1.278} \tag{11}$$

The use of more parameter power law approximations (6) and (7) does not change the RMSE value as well as the correlation. This was not expected. The polynomial approximations for z (8) is much less accurate and the same approximation for Z expressed in [dBZ] (9) is even worse. 4182 one-minute DSD spectras were used for this test. The simple power law approximation seems to be the best one.

5 Result 2: comparison of Z-R approximations using different DSD types

5.1 Selected methods distinguishing the rain type

Here we choiced very simple methods distinguishing two rain (DSD) types more or less mathematically.

5.1.1 Tokay-Short method

This method was presented in [Tokay, A.; Short D., 1996]. The authors defined a CS parameter:

$$CS = \frac{N_o}{4.\,10^9\,R_g^{-4.3}} \tag{12}$$

where R_g is the rain gauge rain rate in [mm/h] (3) and N_o is the “intercept” parameter of the Gamma DSD approximation having been derived through the moment method. The condition CS <1 (or CS >1) indicates the stratiform (or convective) rain type.

According to the Table 2 for the spectras of CS >1 (1061 spectras), the decreased RMSE announced an accuracy improvement in comparison with Table 1 (situation with no distinguished spectras). Also the correlation is slightly better. For the case of CS <1 there is only a negligible improvement (because of mixed spectras).

Table 4. $W > 1$ (1352 spectras) Found parameters of Z-R approximations (5–9) using selected Czech DSD data.

Type	a (or x)	b (or y)	q	RMSE[%]	corr.
(5)	459	1.427	0.0000	8.07	0.85
(6)	449	1.453	0.0919	8.08	0.82
(7)	455	1.415	0.0087	8.08	0.79
(8)	$> 10^{10}$	6.854	−0.3149	118.30	0.00
(9)	0	960.295	32.4439	2671.16	0.85

Table 5. $W < 1$ (2 830 spectras) Found parameters of Z-R approximations (5–9) using selected Czech DSD data.

Type	a (or x)	b (or y)	q	RMSE[%]	corr.
(5)	174	1.357	0.0000	7.96	0.91
(6)	171	1.356	0.0551	7.97	0.92
(7)	172	1.333	0.0094	7.96	0.93
(8)	$> 10^{10}$	5.053	−0.2475	133.58	0.02
(9)	0	176.523	34.2284	2189.97	0.93

Table 6. $R > 2.5$ mm/h (380 spectras) Found parameters of Z-R approximations (5–9) using selected Czech DSD data.

Type	a (or x)	b (or y)	q	RMSE[%]	corr.
(5)	220	1.329	0.0000	8.55	0.71
(6)	474	0.326	0.6909	8.50	0.72
(7)	317	0.956	0.0220	8.50	0.72
(8)	$> 10^{10}$	1.432	−0.0309	85.07	0.00
(9)	NA	NA	NA	NA	NA

Table 7. $R < 2.5$ mm/h (3802 spectras) Found parameters of Z-R approximations (5–9) using selected eCzech DSD data.

Type	a (or x)	b (or y)	q	RMSE[%]	corr.
(5)	227	1.263	0.0000	12.39	0.66
(6)	224	1.302	0.0996	12.40	0.66
(7)	223	1.231	0.0512	12.40	0.66
(8)	$> 10^{10}$	13.607	−3.2136	127.69	0.83
(9)	0	237.884	49.7092	2150.70	0.66

5.1.2 Waldvogel method

The author of [Waldvogel A., 1974] found a simple criterion. He defined a parameter $W = R_z/R_g$, where R_z

$$R_z = \left(\frac{\int_0^\infty D^6 N(D)\, dD}{300} \right)^{2/3} \qquad (13)$$

is the theoretical rain rate derived from the radar measurement using the Marshall-Palmer approximation $Z = 300\ R_z^{1.5}$ (the radar reflectivity factor Z was computed in accordance with its definition as the 6th moment of the DSD or it is simply measured by radar) and R_g is the rain gauge rain rate. If $W < 1$ (or $W > 1$) then the small (or large) rain drops are prevailing in the DSD compared to the situation in the average rain being described by the Marshall-Palmer relationship (i.e. Z=300 $R^{1.5}$). In European region there the presence of larger rain drops can help to estimate the rain type.

It is seen from both Tables 4 and 5, that the Z-R approximations fit much better for two separate types of rain (i.e. DSD) being distinguished in accordance with the Waldvogel criterion. The RMSE has decreased from 12 to 8% roughly in comparison with the situation using all spectras (Table 1). Also the correlation is obviously higher.

5.1.3 Rain rate method

We made an assessment of the numerical threshold for DSD of the rain rate above 2.5 mm/h (380 spectras). If the rain rate exceeds this threshold, there is the probability of convective type of DSD. So we have a very rough criterion. From Table 6 we can see an accuracy improvement, practically it means that the Z - R relations derived from spectras corresponding to the rain rates above 2.5 mm/h are more of the similar type and the RMSE of Z-R approximations decreased from 12% to the value of about 8.5%. The spectras for rain intensity below 2.5 mm/h (3802 spectras) are not stable and the accuracy of the Z-R approximation was not improved.

6 Conclusion

This study should show two results. The first one is the comparison of an accuracy of 5 types of the Z-R approximation. The accuracy is measured through the RMSE as well as the correlation coefficient. There are no doubts that the simple and classical two parameters power-law relation fits as the best one. It is even better than the „improved“ three-parameter approximation. The second order polynom seems to be a no appropriate approximation.

The second result is the presented study of the Z-R relationships corresponding to the different DSD types, which were defined mathematically. The accuracy improvement is obvious especially when distinguishing the DSD types in two classes according to Waldvogel criterion.

Acknowledgements. This contribution was supported by the grant Nr. 102/01/0029 of the Czech Grant Agency and by the MŠMT Grant OC280.002 supporting the COST280 project.

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