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Correction of attenuated C-band reflectivity fields by means of a variational method

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Abstract. Attenuation is one of the most important problems for precipitation estimation at wavelengths shorter than 10 cm. However, for most of the existing radars no correction for attenuation is implemented due to the instabilities of the inversion equation, already shown by Hitschfeld and Bordan in 1954.

The present study proposes a methodology based on the minimization of a cost function that imposes the corrected field to be “close” to the analytical solution and, at the same time, to satisfy some constraints. One of these constraints comes from the orographic returns, assuming that the Path Integrated Attenuation may be estimated as the difference between clear-air and rainy conditions (similarly to the Mountain Reference Technique).

The performance of the methodology has been tested in the case of C-band radars under some simplifying assumptions and corrected fields are validated against the measurements of a network of raingages.

1 Introduction

Precipitation attenuation is a serious problem that makes more difficult the quantitative use of C-band radars (which are the most outside the USA). However, operational radars do not use to apply any correction because of the high degree of instability of the inversion equation derived by Hitschfeld and Bordan (1954).

These authors already showed the difficulties of using their solution if radar measurements were affected by errors (in particular, by small calibration errors), which made them conclude that the correction is useless if these errors cannot be reduced to very narrow limits. As a possible solution, they introduced the idea of using a “gage deep in the storm”, illustrating the need of additional constraints in the correction process.

On the other hand, the need of shorter wavelengths for air- and satellite-borne radars has promoted the development of attenuation correction techniques (using single or dual frequency, single or dual beam and single or double polarization), based on introducing additional constraints to the attenuation model. Most of the single frequency corrections for satellite-borne radars are based on the Surface Reference Technique (see for example Meneghini et al., 1983; Iguchi and Meneghini, 1994; Marzoug and Amayenc, 1994; Amayenc et al., 1996; or Iguchi et al., 2000), which consists on estimating the total attenuation from surface returns. This concept was also implemented by Delrieu et al. (1997) for ground-based radars, using the return of important mountains to constrain the attenuation equation.

Here, a methodology to correct for attenuation (mainly in C-band radars) is presented. It consists on the minimization of a cost function, that includes one term accounting for the attenuation model and another for the constraints. In this study, we implemented the concept of the Mountain Reference Technique to stabilize the attenuation equation.

2 Attenuation equations

From the radar equation (see for example, Doviak and Zrnic, 1992, the measured reflectivity can be expressed (in $mm^6 \cdot m^{-3}$) as:

$$Z_m(r) = \frac{P_r(r) \cdot r^2}{C} \quad (1)$$

where $P_r(r)$ is the received power from a range r and C is the radar constant that depends on radar parameters such as peak power, wavelength, antenna gain, beam width, pulse duration,...

Then, the measured reflectivity can be expressed as a function of the non-attenuated (or equivalent) reflectivity in $mm^6 \cdot m^{-3}$, $Z(r)$:

$$Z_m(r) = Z(r) \cdot A(r) = Z(r) \cdot \exp\left(-0.2 \cdot \ln(10) \cdot \int_0^r k(s)\ ds\right) \quad (2)$$

where, $A(r)$ is the attenuation term, which can be expressed as the integral of the specific attenuation in $dB \cdot km^{-1}$, k, along the path of the radar beam.

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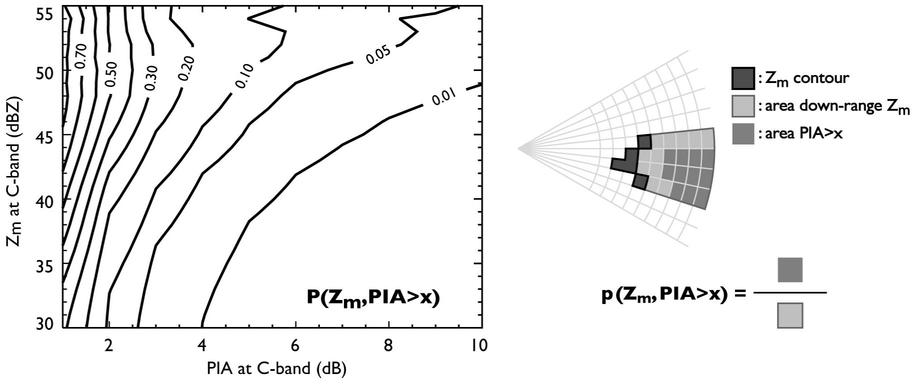

Fig. 1. Isolines indicating the portion of area down-range of a measured reflectivity pattern (Z_m–y-axis) that is affected by a path-integrated attenuation exceeding the PIA value displayed in the x-axis.

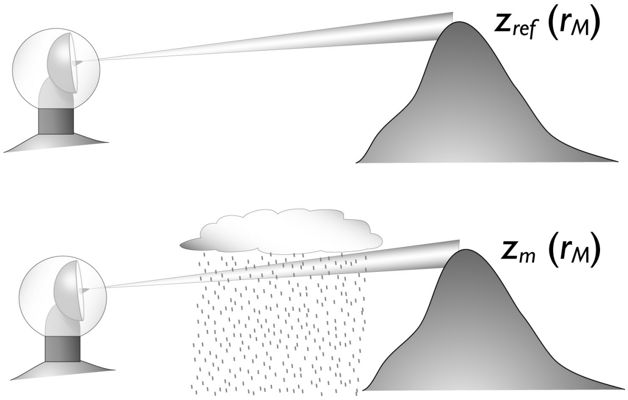

Fig. 2. Scheme of the Mountain Reference Technique. The total attenuation at mountain ranges is estimated as the difference between the mountain return in clear-air conditions and the measured reflectivity in rainy conditions.

Assuming that the specific attenuation can be related to $Z(r)$ by a power-law,

$$k\,(r) = \alpha \cdot Z\,(r)^{\beta} \quad (3)$$

Hitschfeld and Bordan (1954) inverted (2) as (see the development of the current expression in Meneghini, 1978):

$$Z\,(r) = \frac{Z_m\,(r)}{\left[1 - 0.2 \cdot \ln(10) \cdot \beta \cdot \int_0^r \alpha \cdot Z_m\,(s)^{\beta}\ ds\right]^{\frac{1}{\beta}}} \quad (4)$$

The form of equation (4) is the cause of the divergence of attenuation correction with small errors in the radar measurement (that might produce the denominator to be close to 0).

3 Attenuation effects at C-band

Some references have analyzed the effects of attenuation at X- and C-band. Particularly, Duncan et al. (1991) found that the signal attenuation at C-band caused 20% underestimation of accumulated rainfall and a maximum of 5% area reduction in a simulation experiment carried out with 6 events that affected the Montreal area, and estimating the attenuation from the measurements of a S-band radar. In a similar analysis in a Mediterranean area, Delrieu et al. (2000) obtained more than 5% of rain rate profiles exceeding a Path Integrated Attenuation (PIA) of 3 dB at 50 km.

In the present study, a similar simulation experiment has been carried out using one year of data measured by the McGill S-band radar. Attenuated reflectivity profiles have been generated from radar measurements (which are supposed to be unaffected by attenuation) with Eqs. (2) and (3), using a fix $Z-k$ relationship (reported in Battan, 1973).

Examples of the results (presented in Fig. 1) are that the 10% of the area down-range a pattern of Z_m=30 dBZ is affected by a PIA exceeding 2 dB, or the 30% of area behind an area of Z_m=50 dBZ being underestimated at least 3 dB. These attenuation values confirm the importance of this phenomenon even at C-band.

4 Proposed methodology

The philosophy of the proposed correction methodology is based on a variational method that consists on the minimization of a cost function composed of two parts: the background term (supplied by the model), $J_b(x)$, and the observations term, $J_o(x)$.

$$J\,(x) = J_b\,(x) + J_o\,(x) = (x - x_b)^T\, B^{-1}\,(x - x_b) + (y - H\,[x])^T\, R^{-1}\,(y - H\,[x]) \quad (5)$$

In the cost function given by Eq. (5),

x: state vector.
x_b: *a priori* estimation of the state vector, given by the model.
y: observations vector.
H: observations operator, that generates the value $H[x]$ that the observations would take if both they and the state vector were perfect.
B: covariance matrix of the background errors $(x_b - x)$.
R: covariance matrix of the observation errors $(y - H[x])$.

In our case, the background term (first summand of Eq. 6) is given by the attenuation Eq. (2), while the other two summands impose corrected reflectivity field to be close to the

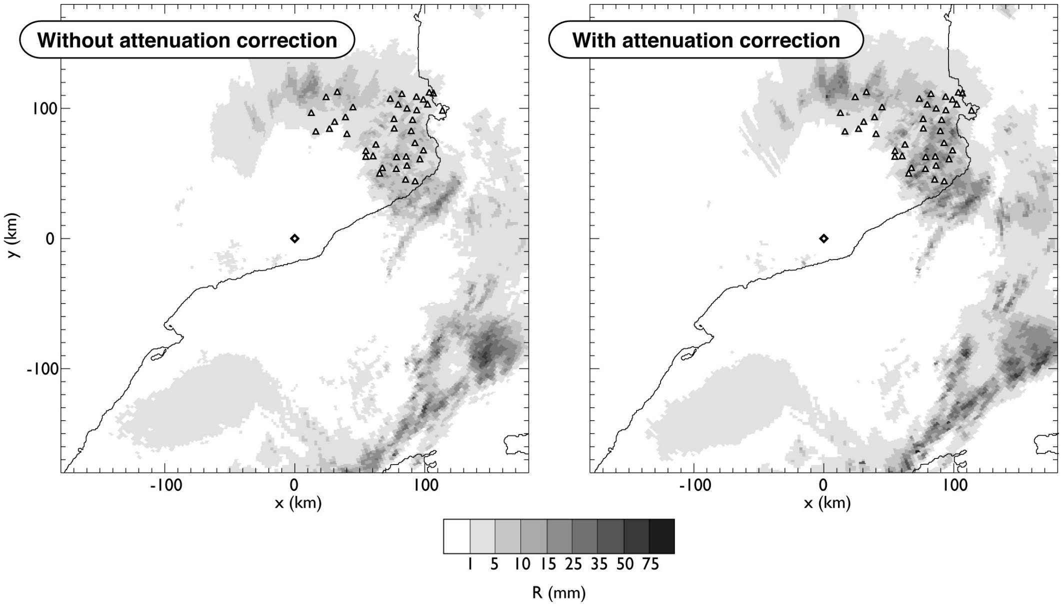

Fig. 3. Hourly accumulated precipitation field estimated from radar scans without (left) and with (right) attenuation correction. Triangles indicate the position of the raingages where radar estimates have been compared against gage measurements (see Fig. 4).

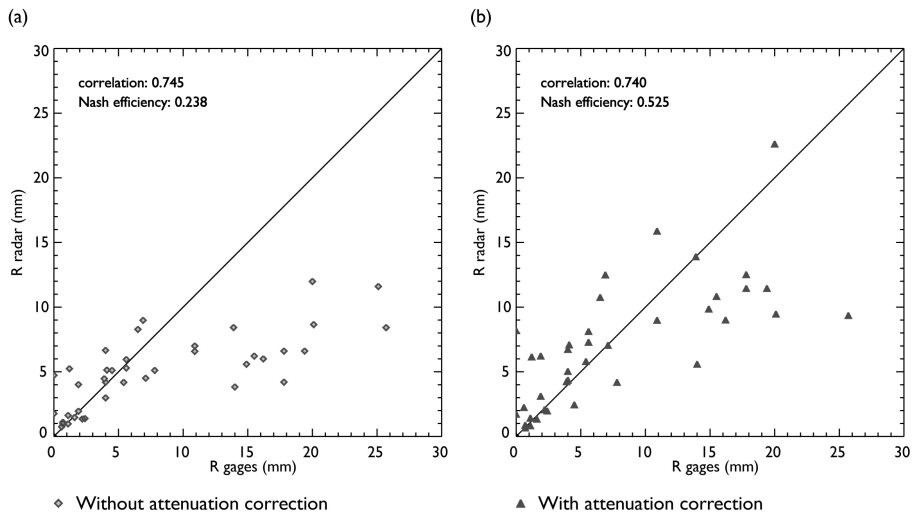

Fig. 4. Scatter plots comparing hourly accumulations in the raingages network shown in figure 3 against radar estimates before correcting radar scans for attenuation (left) and after (right).

constraints. In a first formulation, the covariance matrices B and R have been considered diagonal. Working in polar coordinates (r_i,θ_j) and expressing all variables in dB (notice they are written with small letters), the implemented cost funcion takes the following form:

$$J\left(\Delta c, z\right) = w_1 \cdot \sum_{i,j=1}^{n_{bins}} \left[z\left(r_i, \theta_j\right) - a\left(r_i, \theta_j\right) - z_m\left(r_i, \theta_j\right) + \Delta c\right]^2 +$$

$$w_2 \cdot \sum_{i,j=1}^{n_{clutter}} \left[z_{ref}\left(r_i, \theta_j\right) - z_m\left(r_i, \theta_j\right) - a\left(r_i, \theta_j\right)\right]^2 +$$

$$w_3 \cdot \sum_{i,j=1}^{n_{bins}} \left[z\left(r_i, \theta_j\right) - z\left(r_i, \theta_{j+1}\right)\right]^2 \qquad (6)$$

In the present formulation, retrieved state variables are the radar calibration error, Δc, and the corrected reflectivity field, z.

The first summand of equation 6 imposes z to be close to the attenuation model (a is the two-way PIA obtained with equation 2), taking into account that the meaurements may be affected by a calibration error, Δc.

The second summand compares the total attenuation at mountain ranges estimated with the Mountain Reference Technique, as the difference between the mountain return in clear-air and rainy conditions ($\hat{a}\left(r_M\right) = z_{ref}\left(r_M\right) - z_m\left(r_M\right)$ see Delrieu et al. (1997) and Fig. 2), against the total attenuation resulting from the attenuation Eq. (2).

Finally, the third term of Eq. (6) imposes contigous azimutal profiles to be close. This is a smoothing constraint, already proposed by Vignal et al. (2003) with the purpose of stabilizing the attenuation correction.

5 Case study

As preliminary results, a case study is presented in Figs. 3 and 4. The proposed technique has been implemented to correct one hour of data measured in September 1999 by the Barcelona INM C-band radar during a convective event. Before the attenuation correction, radar scans have been corrected for mountain screening effects. Finally, for the generation of accumulated precipitation fields (see Fig. 4), ground clutter identification and substitution have been also carried out, according to the technique presented by Sánchez-Diezma et al. (2001).

The effect of attenuation caused by precipitation appears clearly in Fig. 4a: radar precipitation estimates present a significant bias as compared against raingage measurements (see Fig. 4a). However, once the proposed methodology has been implemented, this bias is almost totally removed with a non-significant increase of the scatter as shown in figure 4b.

The technique has also estimated a positive calibration error, which has been removed in the corrected field (this explains lower accumulation values in some areas of the corrected field).

6 Conclusions and future work

Although attenuation is a serious problem for precipitation estimation at C-band, most of the operative radars do not use to apply any correction due to the high degree of instability of the inversion equation. To avoid these instability problems, most of the existing techniques introduce additional constraints to the attenuation equation.

In this study, a methodology for attenuation correction based on the minimization of a cost function has been presented. In the present formulation, total attenuation at important mountain ranges is estimated similarly to the Mountain Reference Technique. These total attenuation estimates are introduced into the cost function to constrain the attenuation model jointly with a smoothing condition.

Preliminary results are also presented showing that, for the studied case, the proposed technique has been able to almost totally remove the bias introduced by attenuation into the radar precipitation estimates.

As the more constraints the more stability of the attenuation correction, one of the principal interests of the presented methodology is the facility with which additional constraints can be introduced into the cost function.

Finally, estimation of the error covariance matrices and the study of the influence of the Drop Size Distribution (that determines both the $Z-R$ and the $Z-k$ relationships) into the performance of the presented technique are the main goals of future work.

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